Corporate Provision of Public Goods*

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Abstract

Firms spend considerable amounts on socially responsible business practices. These expenditures are often seen as an indirect form of profit maximization or a perquisite of the manager at shareholder expense. We offer a more direct explanation. Since shareholders enjoy both consumption and private benefits from public goods, managers acting on their behalf will (1) provision more public goods than decentralized shareholders would and (2) always produce less than the profit maximizing output. Under mild conditions, (3) the firm produces the socially optimal quantity and partially provisions the public good, without intervention by a social planner, and (4) decreasing marginal production costs increases public goods as much as decreasing marginal externalities. The results are robust to endogenous formation of socially responsible firms and the possibility of takeover by a profit maximizing outsider.

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1 Introduction

Being good is expensive. In 2007, a survey of 155 US firms reported philanthropic spending (donations of cash and products) amounting to \$11.5 billion.[1] Indirect costs like environmentally friendly manufacturing, employee volunteering, and favoring higher cost suppliers who offer better working conditions add considerably to the bill. In this paper, we examine the motivations of and outcomes produced by socially responsible firms.

In *The Social Responsibility of Business is to Increase Profits* (1970), Milton Friedman argued that such expenditures contradict a manager's duty to shareholders, stating: (*i*) "The manager is the agent of the individuals who own the corporation... and his primary responsibility is to them." (*ii*) "Insofar as his actions in accord with his 'social responsibility' reduce returns to stockholders, he is spending their money." (*iii*) "The stockholders... could separately spend their own money on the particular action if they wished to do so. The executive is exercising a distinct 'social responsibility,' rather than serving as an agent of the stockholders... only if he spends the money in a different way than they would have spent it."

Friedman suggests that socially responsible actions by a firm (beyond legal requirements) reflect only one of two distinct things: either (1) the firm is maximizing profits in a sophisticated way, or (2) managerial incentives are misaligned with shareholder interests. Although he acknowledges that shareholders may value both profits *and* 'social goods,' Friedman concludes shareholders provision the latter more efficiently than the firm; hence there is no reason to delegate these responsibilities to a manager.

Most subsequent literature on the subject attributes such expenditures to indirect profit maximization. For example, Porter and Kramer (2002) as well as Besley and Ghatak (2007) argue that firms "sacrifice" profits in order to differentiate their products for consumers that will pay premiums (in excess of these costs) for goods or services produced in a socially responsible manner. Some economists claim that shareholders of socially responsible firms feel a behavioral "warm-glow" just like individuals who personally give to social causes. For

[1] Committee Encouraging Corporate Philanthropy, "Giving in Numbers: 2008 Edition."

example, Zivin and Small (2005) show that, owing to taxation of corporate profits, owning shares in a socially responsible firm is more efficient than personal giving for philanthropic minded individuals. In other words, if firms can provide "warm-glow" to shareholders more cheaply than individuals can purchase it from the market, then firms should provide it. Whether socially responsible businesses indirectly maximize profits or shareholders offset personal giving with firm provided "warm-glow," the net effect is the same—shareholders end up with more money in their pockets.

The other branch of the literature suggests that these expenditures stem from agency problems. Baron (2007) argues that corporate philanthropy harms shareholders only if it is an imperfect substitute for personal giving, and if shareholders are surprised by such philanthropy, as shareholders who do not benefit from such giving will divest. Alternatively, many social scientists outside economics see no dilemma in misaligned managerial and shareholder incentives—firm management should not work just for shareholders but for society at large (see Garriga and Mele 2004 for a survey).

Our paper falls into neither branch. Instead, we show that perfect alignment of managerial incentives to shareholder interests actually leads to socially responsible firms. The model contains no frictions, such as taxes or agency problems, nor behavioral factors, such as warm glow, nor indirect means of converting giving into profits. Nonetheless, shareholders direct the manager to sacrifice profits, both directly to philanthropy and indirectly by reducing output (and the resulting harms from production). Shareholders end up happier but poorer.

How can this be? Critically, shareholders care both about consumption and their *personal gains* from the public good; that is, we adopt the familiar "pure altruism" framework widely used in public economics.[2] This creates scope for the manager to provide public goods if he can do so more efficiently than shareholders on their own.

The first such efficiency comes from centralized giving—the manager can play a key com-

[2] We use the term *pure altruism* not in the vernacular sense of "selfless concern for the welfare of others," but rather as economists typically do: individuals have preferences over private consumption and total supply public goods but not over how public goods are funded *per se.*

mitment role on shareholders' behalf. After all, shareholders recognize they face a free-rider problem when it comes to decentralized contributions to some public good—every shareholder would benefit if all contributed to the public good, but each has a private incentive not to contribute. The manager, however, can help solve this problem by provisioning public goods centrally. Our first result shows that incenting manager behavior to reflect shareholders' interest results in: (1) greater provision of the public good than shareholders would undertake when decentralized, and (2) higher shareholder welfare.

The manager's control of production levels (and the resulting negative externality to shareholders) leads to the second efficiency. For instance, to the extent that shareholders care about global warming, a plant that produces greenhouse gases also affects their welfare. Now, shareholders might simply incent the manager to maximize profits and then undo the environmental damage themselves through contributions to sequestration, carbon offsetting and the like. Our second result is that this is never optimal: shareholder welfare always increases by incenting the manager to produce less than the profit maximizing output. To see why, consider the benefits of the last unit of production. The increase in profits is negligible while the welfare reduction owing to the externality is not. While shareholders can spend profits to reverse the externality, it would clearly be more efficient to direct the manager not to produce the output in the first place.

Exactly how much will the manager abate production? Suppose a social planner ran the firm for the benefit of all citizens rather than just firm shareholders, selecting both the production quantity and the disposition of profits. The socially optimal output would equalize marginal profit and the marginal cost of negating the damage—for at lower production levels, cleanup costs grow slower than profits, while the opposite holds for higher production levels. Our third, and main, result shows that if, at the socially optimal production level, shareholders would benefit from any contribution to the public good at all, then a manager, maximizing shareholder interests, produces exactly the socially optimal quantity without any intervention by a social planner.

To summarize, when managerial incentives reflect shareholder preferences, the firm plays a positive role in ameliorating the free-rider problem among shareholders. More strikingly, the firm also plays a positive role for society at large. We identify conditions where the firm produces socially optimal output, even though the manager only cares about shareholders.

The model also offers some striking policy comparisons. In choosing between the development of a cleaner production technology or a more efficient one, a social planner concerned about pollutants would naturally lean toward the former. In contrast, we obtain a neutrality result—both technological innovations will result in the same overall level of pollution/public goods by the socially responsible firm.

Our findings help rationalize the trend toward socially responsible behavior by firms. This trend coincides with greater manager accountability and greater shareholder input in determining the goals of a manager. For example, the percentage of outside board directors has steadily risen since the early 1970s (Hermalin and Weisbach, 1988; Borokhovich *et al.*, 1996, Table 1; Dahya and McConnell, 2005). Similarly, board driven CEO turnover has increased since the early 1990s (Kaplan and Minton, 2012). The SEC's adoption of Rule 14a-8 (the shareholder proposal rule) in 1943 opened the way for shareholder activism, but the rise of activist institutional shareholders, especially pension funds in the mid 1980s and more recently hedge funds, has increased shareholder oversight of firm management (Gillan and Starks 2000). Very recently, laws like Sarbanes-Oxley Act (2002) and Dodd-Frank Wall Street Reform (2010) in the US and Shareholders Rights Directive (2007) in the EU have been enacted to increase management accountability to shareholders. In terms of the model, the vast and increasing contributions of firms toward public goods, the greater sensitivity of firms toward regulating the practices of input suppliers to ensure fair wages and healthy working conditions, and the trend toward self-regulation all reflect greater responsiveness to shareholders rather than agency problems whereby the manager places his or her interests above those of shareholders.

To make the intuition as transparent as possible, the baseline model is of a monopolist owned by exogenous, identical shareholders. To highlight the robustness of the intuitions, we then enrich the model with a number of more realistic features. We endogenize shareholding and show that a socially responsible firm can arise even when shareholders are *ex post* worse off than those choosing not to finance the firm. The key here is that *ex ante* gains to financing the firm attract investors, but resulting public goods benefits spill over to non-investors *ex post*. Could such a firm, though, survive a takeover attempt by a pure profit maximizer? We show that even when the potential acquirer has all the bargaining power, it cannot profitably acquire the socially responsible firm. No more shareholder utility can be extracted from the firm—the manager runs it as efficiently as possible from their perspective. Thus, any buyout would have to be unprofitably subsidized by the takeover artist.

We also allow for non-identical shareholders. Here, the results are more mixed. When shareholders are heterogenous, some may not delegate all voluntary contributions to the manager, though some do. Moreover, we show that it is impossible, in general, to determine whether heterogeneity makes the conditions for socially optimal production easier or harder to satisfy.

Finally, we place the firm in an imperfectly competitive context. The firm no longer chooses the socially optimal output. Since firms do not internalize the effects of own output on the profits or the pollution on competitor decisions, the alignment between firm and social incentives breaks.

The remainder of the paper proceeds as follows: Section 2 describes the model. Section 3 derives the our main results. In section 4, we examine several policy comparisons through the lens of the model. Section 5 complicates the model in various ways to illustrate the robustness of the intuition. Finally section 6 concludes. We relegate more technical proofs to an appendix.

2 Model

A firm produces and sells an amount q of a product. Production generates $\pi(q)$ profits, where $\pi(\cdot)$ is concave and single-peaked. Thus, there exists a unique profit maximizing quantity $\widehat{q}$ (i.e. $\pi'(\widehat{q}) = 0$). Production, however, also depletes a public good. The replacement cost of this public good is $\psi(q)$, which is strictly increasing and convex. By producing nothing, a firm earns no profits, but neither does it deplete the public good (i.e. $\pi(0) = \psi(0) = 0$); however, the first unit of production generates more profit than the replacement cost of the public good (i.e. $\pi'(0) > \psi'(0)$).

The firm is owned by n identical shareholders, each with strictly convex preferences represented by the utility function $u(c, g)$, increasing in both private consumption c and public goods quantity g. (Later we will relax the assumption that shareholders are identical.) Of course, not everyone in society is a shareholder. There are also $N - n$ potentially heterogeneous non-shareholding citizens. To isolate the effects of the firm's actions on welfare, we assume that these citizens have neither wealth nor do they receive dividends from the firm. They do, however, benefit from the public good—non-shareholding citizen i has utility $v^i(g)$, increasing and strictly concave in its argument.

We consider a three stage game. First, the shareholders meet to determine the manager's contract. Next, the manager simultaneously chooses the production quantity q and an amount α to contribute to the public good. The remaining profits are distributed equally among the shareholders. Finally, each shareholder simultaneously contributes an amount, β_i, to the public good. Thus, the level of public goods provided is

$$g = -\psi(q) + \alpha + \sum_{j=1}^{n} \beta_j$$

Because ψ captures depletion of the public good in monetary units, features, such as increasing cost to provision the public good as damage increases, are implicitly included. After contributing to the public goods, all remaining cash is consumed by the shareholder. Thus, we can write each shareholder's utility as function of q, α, and β_i

$$u\left(\frac{\pi(q) - \alpha}{n} - \beta_i, -\psi(q) + \alpha + \sum_{j=1}^{n} \beta_j\right) \tag{1}$$

Citizen i's utility can be similarly stated

$$v^i\left(-\psi(q) + \alpha + \sum_{j=1}^{n} \beta_j\right) \tag{2}$$

What determines the manager's payoffs? In the spirit of Friedman, we suppose that the manager's incentives align his payoffs with those of shareholders. The contract written for the manager induces him to select q and α to maximize the utility of a representative shareholder. Thus, the manager's objective function is

$$\max_{0\leq q, 0\leq \alpha \leq \pi(q)} u\left(\frac{\pi(q)-\alpha}{n}-\beta_i, -\psi(q)+\alpha+\sum_{j=1}^{n}\beta_j\right)$$

Formally, shareholders specify a forcing contract dictating q and α. The remainder of the analysis examines the properties of such contracts.

It is important to note that the manager is not simply a social planner in disguise. He seeks to maximize the payoffs of shareholders rather than society at large. Thus, the preferences of the $N-n$ non-shareholders do not figure in the manager's objective function as they would for a social planner. In general, the manager and the social planner would disagree about the optimal contributions to the public good.

Before proceeding, one must specify the behavior in the voluntary contributions game following the choice of (q, α) by the manager. The following lemmas show that there is a unique equilibrium and furthermore all shareholders contribute the same amount (if any) in this equilibrium. The proofs of these lemmas are contained in an Appendix.

Lemma 1 *Following every (q, α), the voluntary contributions game has a unique, symmetric equilibrium.*

Since there is a unique equilibrium following every choice of (q, α), we can speak unambiguously about the manager's problem accounting for the subsequent strategic interaction of shareholders. We begin our analysis by showing that, for a fixed amount of output, there is a unique contribution α that maximizes the manager's objective function.

3 Analysis

Fix the manager's choice of output at q, and consider the contributions of the manager and shareholders. When shareholders contribute strictly positive amounts to the public good (i.e. $\beta_i > 0$ for all i) absent any contributions from the manager, do shareholders benefit by delegating public goods contributions to the manager? The following lemma suggests that answer is no.

Lemma 2 *When shareholder contributions are interior (i.e. $\beta_i > 0$), manager contributions per shareholder α_i (where $\alpha_i = \alpha/n$) crowd out private contributions at a one for one rate. That is,*

$$\frac{d\beta_i}{d\alpha_i} = -1$$

The proof of the lemma follows directly from Bergstrom, Blume, and Varian 1986, Theorem 6, part (i), p. 42.

Kolm triangles (1970) graphically illustrate the intuition. These figures are the equivalent of Edgeworth boxes in an economy with one private and one public good. Figure 1 depicts the situation where a firm owned by two shareholders provides no public goods—all public goods contributions are decentralized.

Every point in the triangle represents a feasible allocation: the vertical distance measures contributions to the public good, and distance to the left and right legs of the triangle measure shareholder 1's and shareholder 2's private consumption respectively. For example, at allocation E, the lengths of $\overline{EO}$, $\overline{EC1}$ and $\overline{EC2}$ represent public goods, private consumption of shareholder 1 and private consumption of shareholder 2 respectively.

The dashed lines represent the feasible choices of consumption and public goods for each player beginning from the situation where no one contributes to the public good. For instance, $\overline{OP1}$ spans the feasible consumption and public goods allocations that shareholder 1 can achieve by privately contributing to the public good. Given shareholder 2's contribution, shareholder 1 will choose an allocation point that is tangent to her indifference curve. For example, when shareholder 2 contributes up to the point $G2$, then dotted segment $\overline{G2P1'}$ depicts shareholder 1's feasible allocations. The point E represents a tangency point for shareholder 1.

An equilibrium occurs when both shareholders' indifference curves are tangent to their budget set at the same allocation—allocation E represents an equilibrium. While this allo-

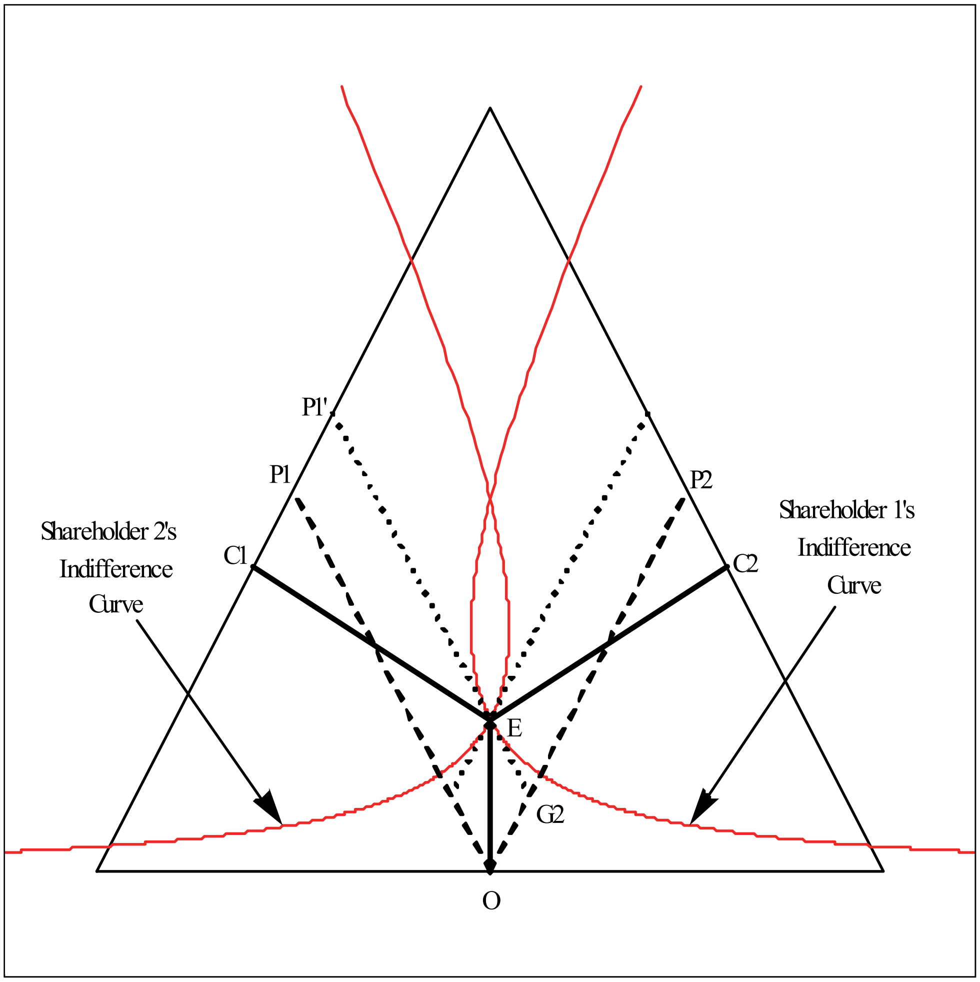

Figure 1: **Equilibrium private giving to the public good in the absence of managerial contributions.**

cation is individually optimal, the familiar free-rider problem remains—any allocation in the lens created by the overlapping indifference curves would be Pareto improving. We highlight this lens in Figure 2.

Now, suppose the manager can contribute to the public good directly. Figure 2 amends Figure 1 by adding vertical segment $\overline{OPM}$, which spans the feasible set of direct managerial contributions to the public good. When manager contributes modestly, i.e. along $\overline{OE}$, the resulting allocation remains E. The starting point on the vertical axis at which individuals begin making private contributions adjusts upward, but since shareholders compensate, the

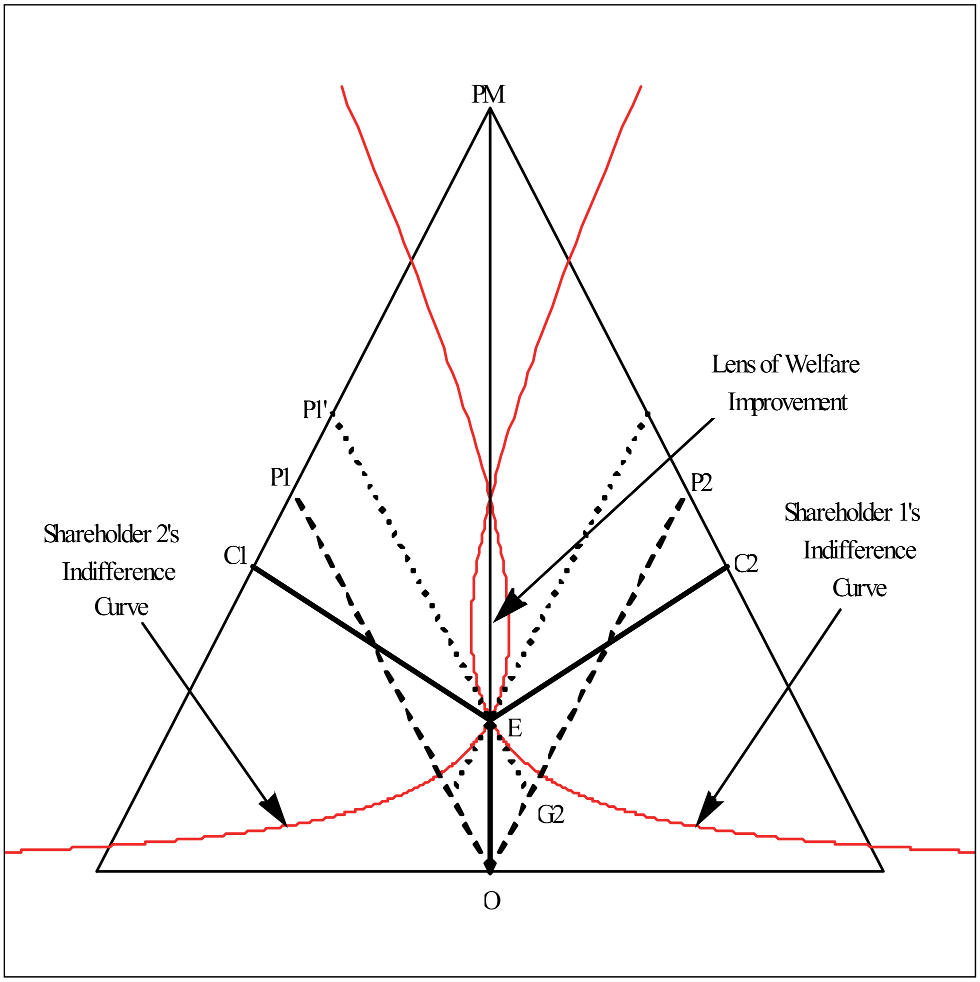

Figure 2: **Lens for Pareto improvements under private contributions to the public good.**

final allocation is unchanged.

This suggests that there is no benefit to having the manager contribute to the public good: firm contributions are exactly offset by reductions from shareholders. The result is familiar in public economics. Similar "neutrality" results hold under a wide array of policy interventions (see, e.g. Warr, 1983; Kemp, 1984; and Bergstrom, Blume and Varian, 1986).

But, the manager is not restricted to contribute on $\overline{OE}$; he can contribute anywhere along $\overline{OPM}$. In particular, the manager can feasibly choose an allocation inside the lens of welfare improvement. Since the manager maximizes shareholder welfare, the manager will continue to contribute to the public good, so long as the lens exists.

The endpoint of this process is shown in Figure 3, where the manager has contributed $\overline{OE'}$ to the public good. As in Figure 2, the vertical line $\overline{OPM}$ represents the set of feasible public goods allocations by the manager. Since the manager maximizes shareholder utility, he will choose the allocation E'. Shareholders' indifference curves are tangent at this allocation—if they were not, the lens would still exist and the manager could improve shareholder welfare by increasing contribution.

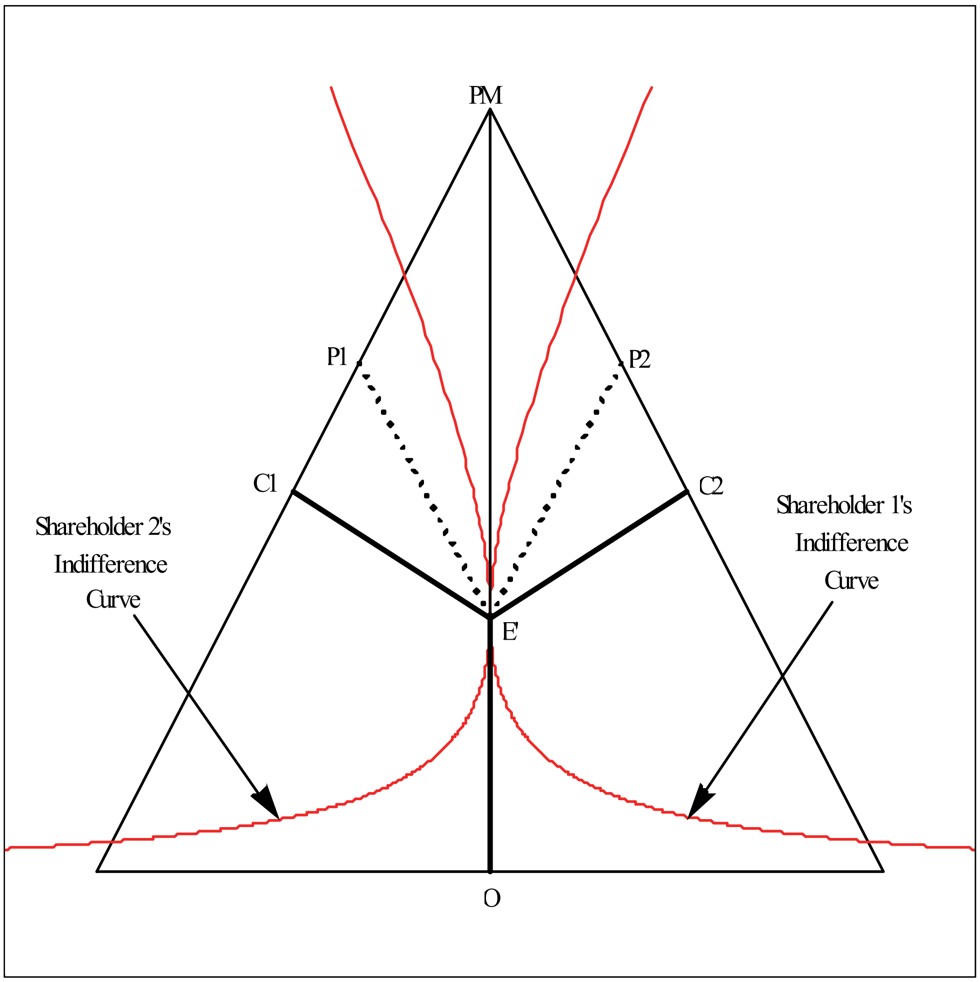

Figure 3: **Optimal managerial contributions to the public good for a fixed level of production.**

Of course, shareholders are free to contribute on their own following the manager's contribution. Segment $\overline{P1E'}$ depicts shareholder 1's feasible allocations when shareholder 2 contributes nothing. Notice from shareholder 1's indifference curve that she wishes to make no additional contributions, and likewise for shareholder 2. Thus, point E' represents the equilibrium allocation. Here, the length segment $\overline{C1E'}$ represents shareholder 1's resultant private consumption and similarly $\overline{C2E'}$, the private consumption of shareholder 2. Notice that the manager's power to commit each shareholder to overcontribute the public good completely crowds out private giving and exhausts all welfare gains.

The graphical intuition is quite general, as the following proposition shows:

Proposition 1 *(i) No shareholder contributes privately—all contributions are delegated to the manager. (ii) Overall public goods provisioning and shareholder welfare are higher relative to the case where the manager is barred from contributing.*

Proof. Suppose to the contrary that each shareholder contributes an amount $\beta_i > 0$ to the public good while the the manager optimally contributes $\alpha' \geq 0$. We will show that the manager can profitably deviate by increasing public goods provisioning and shareholder welfare such that shareholders optimally delegate all contributions to the manager. Let $c(q, \alpha', \beta_i')$ and $g(q, \alpha', \beta_i')$ be the levels of consumption and public goods enjoyed by each shareholder respectively. If instead, the manager contributed $\alpha'' = \alpha' + n \times \beta_i'$ (i.e. an additional β_i' per shareholder), then by Lemma 2, shareholders would optimally respond by contributing nothing and the overall level of the public good would be unchanged; that is, $c(q, \alpha'', 0) = c(q, \alpha', \beta_i')$ and $g(q, \alpha'', 0) = g(q, \alpha', \beta_i')$. Now suppose that the manager increased α'' slightly. Since individual contributions are at a corner solution, Lemma 2 no longer applies. Instead, each shareholder would experience a decrease in utility of $u_c(c(q, \alpha', \beta_i'))$ and an increase of $n \times u_g(g(q, \alpha', \beta_i'))$ from increased public goods provision. But since β_i' was optimal originally, then $u_c(c(q, \alpha', \beta_i')) = u_g(g(q, \alpha', \beta_i'))$ and hence $u_c(c(q, \alpha', \beta_i')) < n \times u_g(g(q, \alpha', \beta_i'))$. Thus, the manager's deviation increases public goods provisioning and shareholder welfare, but this contradicts the notion that the original contract was optimal. ■

And furthermore, it can be shown (in an Appendix) that for any production level q, the manager's problem in α is well behaved:

Lemma 3 *For every q, the manager's problem has a unique maximand in α.*

Why does centralization help? Because the manager can act as a commitment device on the part of shareholders. Indeed, the manager can perfectly solve the free-rider problem from the perspective of shareholders. Under the optimal contract, the manager fully internalizes the benefits to all shareholders of increased giving to the public good and sets output and contributions accordingly. This obviously relies on the assumption that shareholders are identical; however, as we show in Section 5.3, the manager continues to play a useful role in solving the collective action problem (albeit imperfectly) even when shareholders are heterogeneous.

The intuition that centralized contribution mitigates free-riding may seem equally applicable to charitable non-profits. There is, however, an important difference between a charity and a firm. Management completely controls firm profits until distributed to the shareholders while a charity relies on voluntary contributions from individuals to fund the public good. Of course, these contributions are subject to the free-rider problem and hence the charity cannot replicate the commitment function of the firm. Thus, a charity centralizes contributions less effectively than the firm.

Next we turn to the quantity decision of the manager, one for which an analog outside the firm is more difficult.

Proposition 2 *When incentives are fully aligned, the manager will produce less than the profit maximizing quantity.*

Proof. Suppose a firm produced the profit maximizing quantity $\widehat{q}$. A slight production decrease would create a first order gain in shareholder welfare due to increased public good (i.e. $\psi'(\widehat{q}) > 0$), but no first order loss in profits (i.e. $\pi'(\widehat{q}) = 0$). Thus, some $q < \widehat{q}$ is optimal. ■

Friedman argued that a manager acting in the interests of shareholders would maximize profits and distribute the proceeds for shareholders to do with as they please. Propositions 1 and 2 show that the shareholders incent the manager to do neither—optimal production is below profit maximizing levels and optimal public goods provision is performed by the firm rather than shareholders.

Propositions 1 and 2 do not imply that, from a societal perspective, the free-rider problem is solved. Notice that the manager's incentives account for the positive externality among shareholders but does not account for the positive externality accruing to non-shareholders. Indeed, it follows immediately from Proposition 1 that public goods would be underprovided were society to rely only on the firm. One might suspect that a similar argument could be made about production. The manager accounts for the negative externality of production on shareholders but takes no account of the externality on non-shareholders. Thus, one would expect the firm to overproduce from a societal perspective.

Before exploring this intuition, consider the following benchmark setting: Suppose that a social planner were given *full control of the firm and its profits.* How much would she optimally produce for the benefit of *all citizens*? By reasoning as in Proposition 1, we conclude that shareholders will make no private contributions to the public good. A utilitarian planner would maximize the aggregate not only of shareholder welfare but also that of non-shareholding citizens. That is, the planner will solve

$$\max_{0\leq q, 0\leq \alpha\leq \pi(q)} nu\left(\frac{\pi\left(q\right)-\alpha}{n}, -\psi\left(q\right)+\alpha\right)+\sum_{i=1}^{N-n} v^{i}\left(-\psi\left(q\right)+\alpha\right) \tag{3}$$

To make the problem interesting, we assume that the population of non-shareholding citizens, $N-n$, is sufficiently large that the planner wants to contribute something to the public good at all production levels. Thus, the planner's problem consists of two parts: The planner needs to choose the optimal production level to generate wealth that may then be allocated between consumption and the public good. Given the wealth created, the planner then needs to make an allocation decision. The following lemma deals with the first aspect of the planner's problem, optimal wealth creation via production. The proof, which is routine but tedious, is relegated to the appendix.

Lemma 4 *The socially optimal production level, q^*, is the unique value of q solving $\pi'(q) = \psi'(q)$.*

Lemma 4 is intuitive. It says that the planner produces up to the point where marginal profit equals the marginal cost of repairing the damage to the public good. When choosing the production quantity, the planner simply asks "Will producing another unit generate more profits than it costs to clean up the resultant damage to the public good?" If the answer is 'yes,' then producing the unit and paying to completely negate the damage always makes society better off, regardless of how the planner decides to use the leftover profits from the additional unit. On the other hand, if the answer is 'no,' and the planner would spend any money at all, from any source, on the public good, then regardless of what other decisions the planner may make, the planner can save money and maintain public goods levels by not producing the additional unit. This intuition readily extends to a richer model where the planner also controls factors beyond those directly related to the firm, such as the ability to tax and redistribute income from citizens.

Slightly more formally, suppose that the planner sought to provide an amount $g^* > 0$ of the public good. Provided that sufficient wealth is generated via production, then one way to reach this target is by choosing $q' > q^*$. In that case, the planner would have to divert profits amounting to $\int_0^{q'} \psi'(q)\,dq + g^*$, leaving

$$C = \pi(q') - \left(\int_0^{q'} \psi'(q)\,dq + g^*\right)$$

in consumption to be allocated. If instead, the planner marginally reduced output by an amount ε, profits would fall by approximately $\pi'(q')\,\varepsilon$ while emissions would decrease by approximately $\psi'(q')\,\varepsilon$. Thus, to implement g^* of the public good, the planner would only need to divert $\int_0^{q'} \psi'(q)\,dq + g^* - \psi'(q')\,\varepsilon$ from profits and hence the resulting consumption would be

$$C' = \pi(q') - \pi'(q')\,\varepsilon - \left(\int_0^{q'} \psi'(q)\,dq + g^* - \psi'(q')\,\varepsilon\right)$$

and since $\psi'(q') > \pi'(q')$, this implies that $C' > C$, i.e. greater consumption is now available with no change in the provision of the public good. Hence, q' could not possibly be optimal. A similar argument establishes that any output $q' < q^*$ is likewise not optimal.

Lemma 4 shows that the planner's problem may be decomposed into separate production and allocation decisions. Production quantity is selected to maximize the size of the "pie," accounting for the costs of repairing the public good. The allocation decision determines the fraction of the pie going to consumption versus public goods provision. Thus, setting $q = q^*$ is a necessary but not sufficient condition for achieving the optimal provision of the public good. First-best requires the combination of optimal production *and* optimal distribution of the resulting profits between private consumption and the public good.

With this benchmark in mind, we now turn to production when the manager controls the firm. The following condition is the managerial analog to the "large N" condition for the social planner's problem—it describes circumstances in which the manager wants to contribute positive amounts to the public good.

Condition 1 (Fundability) *We say the public good is* fundable *iff shareholders prefer the manager to contribute* something *to the public good when production is optimally abated. Formally,*

$$u_c\left(\frac{\pi(q^*)}{n}, -\psi(q^*)\right) \leq n u_g\left(\frac{\pi(q^*)}{n}, -\psi(q^*)\right) \tag{4}$$

Otherwise we say the public good is unfundable.

We now come to the main result of the paper: if the fundability condition holds, then the manager and the social planner make *exactly the same* production decision. Formally,

Proposition 3 *(i) The manager chooses the socially optimal quantity q^* iff the public good is* fundable*; otherwise the firm overproduces (i.e. $q \in (q^*, \widehat{q})$). (ii) Furthermore, the manager provisions strictly positive amounts of the public good iff the inequality in equation (4) is strict.*

Proposition 3 (proved in the Appendix) offers tight conditions in which the manager produces the socially optimal level of output. If, at this level of output, shareholders would have the manager contribute anything at all to the public good, then the incentives of the manager are perfectly aligned with societal incentives in terms of output. The intuition is as follows: While the shareholders do not desire the same overall level of the public good as does society at large, they do desire that the public good be provisioned as efficiently as possible by the manager. When the fundability condition holds, shareholders will direct the manager to divert a portion of profits to the public good for any production level above the social optimum. When there is overproduction, the marginal depletion of the public good exceeds the marginal profit. As a consequence, the manager can reduce production and increase dividends while still maintaining the same level of the public good. The stopping point occurs when the two margins are equalized—exactly the social planner's optimality condition. In short, when the fundability condition holds, government intervention is no longer necessary to solve the "missing market" problem of the production externality.

Figure 4 graphically illustrates these intuitions and the link between the planner's and manager's problems. Total public goods levels are measured on the vertical axis, while per shareholder dividends (private consumption) are measured horizontally. The hypotenuse of each right triangle denotes the feasible set of allocations under a different production choice ($q' < q^* < \hat{q}$) by the manager or planner. The upper left corner of a triangle denotes the allocation where all profits are diverted to the public good and the lower left corner, where none are. Thus, the dotted triangle ($q = q' < q^*$) represents the economy when production is less than than the social optimum—the firm pollutes relatively little ($-\psi(q')$), but the maximum possible public goods level under this production choice is also relatively low ($\pi(q') - \psi(q')$). In fact, every production decision up to the social optimum q^* (denoted by a bold triangle) yields a feasible allocation set that strictly dominates that induced by q', because profits are growing faster than the cost of cleaning up the resultant pollution. The situation changes, though, at higher production levels. Between q^* and profit maximizing quantity $\hat{q}$ (denoted by the dashed triangle), the absolute size of the triangles continues to increase—since there are more profits to allocate, the span of the feasible allocation set grows—but since the cost to clean up pollution now grows faster than profits, the set of feasible allocations shift downward on the public goods axis.

The manager's indifference curve is also shown in Figure 4. We have omitted the planner's indifference curves, but when the number of citizens is much larger than the number of shareholders ($N \gg n$) aggregated preferences place nearly all weight on the public good—in this case, the planner's indifference curves could be approximated by horizontal lines. The planner desires the vertically highest allocation possible: he chooses the socially optimal production quantity q^* and allocates all profits to the public good. If the manager's indifference curve has a tangency on the bold triangle representing socially optimal production, then clearly, no other production quantity can produce a superior allocation for shareholders—the existence of such a tangency is the graphical analog of the fundability condition. When the manager's indifference curve is so steep that no such tangency exists (i.e. fundability fails) the manager will produce more.[3] Thus, one can readily see why, if the fundability condition holds, manager and social planner agree on production levels, if not allocation.

How restrictive is the fundability condition? There are several reasons to believe that it is likely to be satisfied in most practical applications. First, if, in the absence of any production, shareholders view the public good as sufficiently important that they would privately contribute positive amounts, then the fundability condition is automatically satisfied. Second, if the decentralized provision of the public good under output $q \leq q^*$ produces any private contributions whatsoever, the fundability condition is also satisfied. Finally, for a fixed dividend per shareholder, there exists a large enough shareholder base n such that the fundability condition is always satisfied.

[3] Note, no tangency may exist on triangles up to that induced by $\hat{q}$ either. Although omitted from Figure 4, triangles for greater q continue to shift downward due to increasing pollution, but their horizontal extent also shrinks, because profits are shrinking. Thus, in such a case, the manager chooses $\hat{q}$ and allocates nothing to the public good (i.e. at the lower right corner of the dashed triangle).

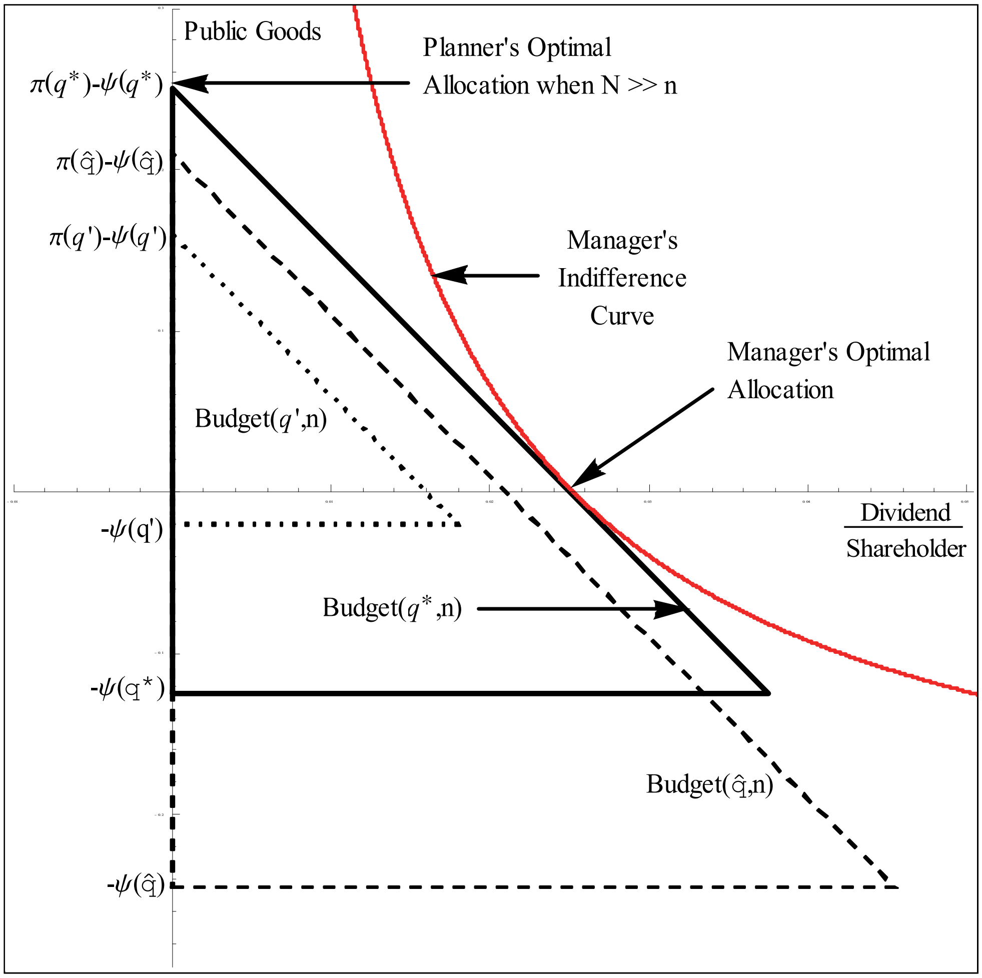

Figure 4: **Comparison of optimal allocation decisions by the manager and a social planner.**

4 Comparative Analysis

In this section, we study how the corporate provisioning of public goods changes in a variety of settings.

4.1 Subsidizing Clean Technology

Proposition 3 suggests that government intervention over production quantity cannot improve production quantity when the fundability condition holds. Nonetheless, governments do take considerable interest in externality mitigating technology—mandating that firms develop or implement technology to reduce externalities like pollution or subsidizing such research directly.

Suppose that the government could choose to subsidize technology development either (1) to make production cheaper or (2) to make it cleaner. Most likely, a government concerned with production externalities would opt for the latter. Indeed, it would seem that subsidizing cheaper production would only exacerbate the pollution problem by inducing the firm to increase output. This, however, ignores the effects of the technology changes on the contract offered by shareholders. The following proposition shows that, when the fundability condition holds, the two investment strategies are *neutral* with respect to public goods provision. Put differently, society may be better off investing in technology which makes production cheaper rather than cleaner, if developing the former technology is less expensive.

Before proceeding, we need to be precise about the technology changes we have in mind. Suppose that under the *cleaner* technology, we replace $\psi(q)$ with $\hat{\psi}(q)$ with the property that, for all q, $\psi'(q) > \hat{\psi}'(q)$. Suppose that under a *cheaper* technology, we replace $\pi(q)$ with $\hat{\pi}(q)$, with the property that, for all q, $\hat{\pi}'(q) > \pi'(q)$. Other than this, $\hat{\psi}(q)$ and $\hat{\pi}(q)$ have the usual properties of profit and externality functions described in Section 2. Finally, to make the two technology improvements comparable in their effectiveness, suppose that, for all q,

$$\psi'(q) - \hat{\psi}'(q) = \hat{\pi}'(q) - \pi'(q) \tag{5}$$

That is, for a given level of output, the cost effectiveness of the cleaner technology is identical to the cost savings from the cheaper technology. To illustrate this, consider the following example:

Example 1 *Define profit and damage repair functions*

$$\begin{aligned} \pi(q) &= \frac{\frac{1}{a^2} - a\left(\frac{1}{a} - q\right)^2}{2} \\ \psi(q) &= \frac{bq^2}{2} \end{aligned}$$

where a parameterizes the cheapness of the production technology and b parameterizes its *cleanliness. The derivatives associated with these functions are*

$$\pi'(q) = 1 - aq$$

$$\psi'(q) = bq$$

Now, if we substitute $\hat{a}$ *for* a *to obtain* $\hat{\pi}(q)$ *and* $\hat{b}$ *for* b *to obtain* $\hat{\psi}(q)$*, then the technology improvements are comparable whenever* $b - \hat{b} = a - \hat{a}$*.*

Our next result establishes the neutrality of comparable technological changes.

Proposition 4 *Suppose the fundability condition holds. Then firm output is* identical *under the cleaner or cheaper technology. Furthermore, total public goods are* identical *under the two technology improvements.*

The Appendix contains the formal proof of the proposition, but the underlying intuition is straightforward. The fundability condition ensures that marginal profit equals the marginal externality, i.e. $\pi' = \psi'$ under the optimal output. Thus, the manager's problem is equivalent to choosing q to maximize $\pi(q) - \psi(q)$. It is helpful to think of this as a familiar firm maximization problem where $\pi(q)$ is a revenue function and $\psi(q)$ is a cost function. Under this view, the cleaner technology is equivalent to a reduction in marginal costs while the cheaper technology is equivalent to an increase in marginal revenues of the same amount. Since the manager only cares about the net of revenues and costs, each of these changes has the *same* effect—a price increase or a marginal cost decrease of \$1 produce the same effect on profits and hence output. Following production, the manager's job is simply to choose between consumption and public goods provision along the budget curve induced by the output decision. Since the relative price, and, indeed, the budget set itself is unaffected by the technological change, the final choice of consumption and public goods provision is also unchanged.

4.2 Widely versus Closely Held Firms

Here, we investigate how changes in the size of the shareholder base affect the corporate provision of public goods. Figure 5 provides a useful basis to examine this question. This figure fixes production and varies the number of shareholders between n and n' where $n < n'$.[4] While, in principle, the production level chosen by the manager might depend on the number of shareholders, provided that the fundability condition holds in both cases, production will remain that same, at q^*. For purposes of this comparison, we will assume that the fundability condition holds under both n and n'. In that case, the manager's decision reduces to simply the choice of α, the amount of profits to spend on public goods. Plotting public goods provision on the y axis and dividends per capita on the x-axis, it is readily apparent that the manager faces a budget line with slope $-n$. Obviously, the more widely held is the firm, the steeper is the budget line.

The manager seeks to maximize the sum of shareholder utility

$$W = nu\left(\frac{\pi(q^*) - \alpha}{n}, -\psi(q^*) + \alpha\right)$$

and, it follows that the indifference curve trading off public goods and dividends per capita is simply

$$\frac{dDiv}{dg} = -\frac{u_c\left(\frac{\pi(q^*)-\alpha}{n}, -\psi(q^*) + \alpha\right)}{u_g\left(\frac{\pi(q^*)-\alpha}{n}, -\psi(q^*) + \alpha\right)}$$

which is independent of the whether the firm is widely or closely held.

Thus, when a firm becomes more widely held, it is equivalent to an increase in the relative price of per capita dividends. As usual, there are two effects: The substitution effect leads the firm to increase its provision of the public good and lower per capita dividends. The income effect depends on whether public goods are normal or inferior goods.[5] Since, in our setting, public goods are normal, the income effect pushes in the opposite direction—away from the increased provision of public goods. Clearly, the net effect can go in either direction depending on the magnitude of the income and substitution effects. To make a determination, one would need to estimate the cross-price elasticity of corporate public goods. If it is positive, widely held firms will provide more public goods; otherwise they will

[4] The bold triangles of Figures 4 and 5 represent identical feasible allocation sets.

[5] Recall that diminishing marginal rates of substitution in a two good setting imply that both consumption and public goods are normal.

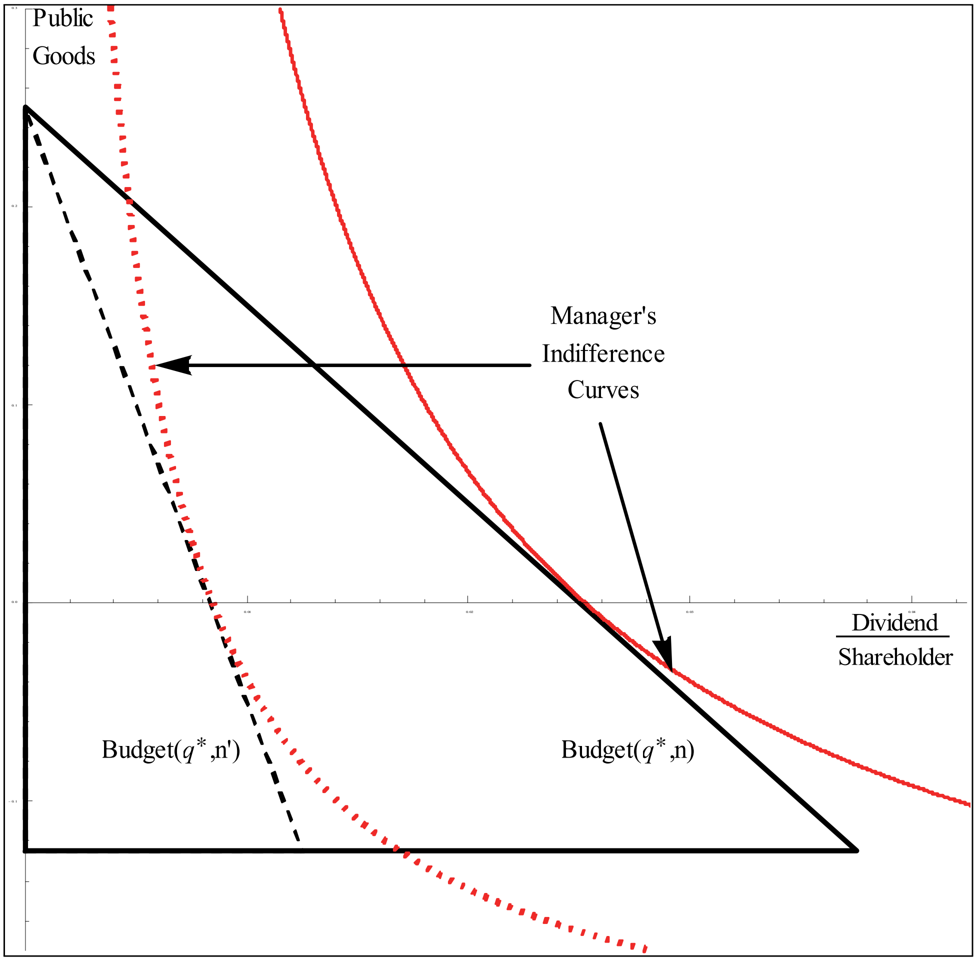

Figure 5: **Change in the manager's optimal allocation as the number of shareholders increases.**

produce fewer. Shareholder preferences depicted in Figure 5 have a 0 cross-price elasticity—here, the manager's allocation is insensitive to the number of shareholders. It is an empirical matter as to the cross price elasticity of corporate public goods. So far as we are aware, no such estimates exist in the literature.

5 Extensions

Up to now, we have restricted attention to a monopoly firm with identical shareholders that required no capital to begin operations. In this section, we explore how our main result, that the firm produces at the socially optimal quantity, q^*, changes when we enrich the model. We also examine how these extensions affect the overall corporate impact on public goods.

5.1 Funding Firms Providing Public Goods

Suppose that, to start up, the firm described in the model requires a one-time infusion of capital for, say, the purchase of the equipment needed to operate. We have shown that, when shareholding is exogenous, shareholders will direct the manager to produce less than the profit maximizing output and contribute to the public good on their behalf. Of course, the benefits from these public goods also accrue to non-shareholders. Thus, when the firm requires an initial investment of capital to set up operations, potential shareholders have an incentive to free-ride by remaining on the sidelines while still enjoying the benefits of the public goods produced.

In this section, we examine whether a socially responsible firm could raise the funds needed to begin operation when shareholding is endogenous. Clearly, if the payoffs from operating the firm at the socially optimal level exceed the outside option, all individuals prefer to own shares of the firm—if the firm were more profitable than outside investments, even when operated in a socially optimal manner, then fund-raising would be easy. At the other extreme, if the firm is so unproductive that it cannot match the performance of outside investments, even when operated to maximize profits, then no rational individual would invest. However, tension arises in the model when the firm earns greater returns than outside investments when operating at the profit maximizing level but lower returns when operated at the socially optimal level. Our main result in this section shows that, when investors are *ex ante* identical, the firm is funded and operated at the socially optimal level. Despite their *ex ante* willingness to fund the firm, *ex post*, these shareholders are worse off than those who stuck with the outside investment.

To formalize this extension of the model, assume the firm must raise n dollars to begin operations. If the firm is funded, shareholders hire a manager (at nominal cost) to do their bidding. Each of $N > n$ identical individuals is endowed with one dollar, which she can invest in either a pollution free bond that pays Π dollars or a single share of the firm. Since we are only interested in the comparison of the two investments, we normalize the returns from the pollution free bond to be $\Pi = 1$. To simplify the analysis while preserving the main economic intuition, we restrict the choices of investors to either investing their entire endowment in the firm or not. We also restrict attention to trembling hand perfect pure strategy equilibria. This latter restriction merely rules out nuisance equilibria where investors coordinate on Pareto inferior non-investment. We could have equivalently utilized a Pareto refinement.

As a benchmark, suppose that there is no opportunity to invest in the firm. In that case, all investments are exclusively in pollution free bonds, the returns from which individuals can voluntarily contribute to the public good. Let $\tilde{\gamma}$ denote the equilibrium contributions by each individual in this situation. We assume that the public good is sufficiently desirable that individuals are at least indifferent to contributing to it, absent the firm. Of course, this is also the situation were the firm to fail in its funding efforts, in which case each investor would obtain the equilibrium payoff $u(1-\tilde{\gamma}, N\tilde{\gamma})$.

Two assumptions capture the interesting situation where the firm is more profitable than the bond when run at a profit maximizing level and less profitable when run at the socially optimal level. The first assumption, which imposes a lower bound on the profitability and damage caused by the firm, guarantees that the firm will be funded. Formally,

Assumption 1. *Capital operated under profit maximization is more desirable than the bond. Formally,*

$$u\left(\frac{\pi(\hat{q})}{n}, -\psi(\hat{q})\right) > u(1,0)$$

where $\hat{q}$ denotes the profit maximizing output.

Since reductions in the public good are costly to individuals, it immediately follows from Assumption 1 that $\frac{\pi(\hat{q})}{n} > 1$. Returns at this level are sufficient to entice individuals to want to become shareholders. Formally,

Proposition 5 *Suppose that Assumption 1 holds. Then, the firm will be funded and operate at the socially optimal level of output.*

We establish the proposition via a series of lemmas. The first lemma says that if contributions to the public good are unchanged when the firm operates at a profit maximizing level, an investor would prefer to become a shareholder rather than invest in the pollution free bond.

Lemma 5 *Suppose that the firm is operated purely to maximize profits, and all individuals contribute $\tilde{\gamma}$ to the public good. Under Assumption 1, individuals prefer to become shareholders rather than to invest in the pollution free bond. Formally,*

$$u\left(\frac{\pi\left(\hat{q}\right)}{n}-\tilde{\gamma},N\tilde{\gamma}-\psi\left(\hat{q}\right)\right)>u\left(1-\tilde{\gamma},N\tilde{\gamma}\right)$$

The proof (in the Appendix) of Lemma 5 uses the Fundamental Theorem of Calculus to show that shifting consumption by $-\tilde{\gamma}$ and public goods by $+N\tilde{\gamma}$ on both sides of the inequality specified in Assumption 1 leaves the ordering intact. The next lemma says that, when the firm is a pure profit maximizer, voluntary contributions to the public good increase compared to the situation where no firm is present.

Lemma 6 *Under Assumption 1, total individual public goods contributions when the firm is funded and run as a pure profit maximizer exceed total individual public goods contributions when the firm is unfunded. Formally,*

$$n\hat{\beta}+\left(N-n\right)\hat{\gamma}>N\tilde{\gamma}$$

The proof (in the Appendix) is straightforward. When the firm is operated at the profit maximizing level, the wealth of shareholders strictly increases while the wealth of non-shareholders is left unchanged. At the same time, the damage caused by firm production reduces the level of the public good absent voluntary contributions. Both effects increase individual incentives to contribute to the public good and hence overall voluntary contributions are higher.

Of course, we have previously shown that shareholders will *not* direct the manager to operate the firm as a pure profit maximizer. Suppose instead, they direct the manager to choose output $\bar{q}$ and contribute $\bar{\alpha}$ to the public good. The next lemma establishes that shareholders enjoy higher utility compared to the case where the firm does not operate. Formally,

Lemma 7 *Under Assumption 1,*

$$u\left(\frac{\pi(\bar{q})-\bar{\alpha}}{n},\bar{\alpha}+(N-n)\bar{\gamma}-\psi(\bar{q})\right)>u(1-\tilde{\gamma},N\tilde{\gamma})$$

where $\bar{q},\bar{\alpha}$ are the optimal production quantity and public goods contribution chosen by the manager and $\bar{\gamma}$ is the equilibrium individual contribution of the remaining individuals who hold bonds.

Together, Assumption 1 combined with Lemmas 5 and 6 imply that shareholders earn higher utility when the firm is operated in a purely *profit maximizing* fashion than when the firm does not operate at all. Obviously, when the manager chooses output and public goods contributions optimally, this only raises shareholder utility further. Since the comparison of payoffs given in Lemma 7 is the relevant comparison when an investor is pivotal in funding the firm (i.e. when $n-1$ others have invested), it then follows that there exists an equilibrium in which the firm will be funded.

Since this is a coordination game, there also exists an equilibrium where investors coordinate on non-investment, but this is ruled out by the trembling hand refinement; hence Assumption 1 guarantees that the firm will be funded. See the Appendix for a proof that an equilibrium where the firm is funded survives the trembling hand refinement.

While Assumption 1 is sufficient for the firm to be funded, it is not necessary. The critical comparison is the ordering given in Lemma 7, which can still hold even if a profit maximizing firm produces less utility than the outside option. Consider the following example:

Example 2 *Suppose that a potential firm has technology specified in Example 1, and investors have quasilinear utility $u(c,g)=c+2\sqrt{1+g}$. Observe that bondholders privately contribute iff public goods levels are negative—in particular, they are precisely indifferent to* *contribution if the firm is not funded (and enjoy $u(1,0) = 3$), and if it is funded, then each bondholder will contribute*

$$\gamma^* = -\frac{\alpha^* - \psi(q^*)}{1 + N - n}$$

iff the public goods level after the firm's actions $\alpha^* - \psi(q^*) < 0$. Knowing this, the firm, if funded, will contribute

$$\alpha^* = n^2 - (N - n)\gamma^* - (1 - \psi(q^*))$$

Suppose $a = \frac{1}{4}$, $b = \frac{1}{16}$, $n = 6$, and $N = 11$. Then

$$\hat{q} = 4, \pi(\hat{q}) = 8, \psi(\hat{q}) = 1, q^* = \frac{8}{3}, \pi(q^*) = \frac{70}{9}, \psi(q^*) = \frac{4}{9}$$

Furthermore $\alpha^* = \frac{4}{9} = \psi(q^*)$ and $\gamma^* = 0$; i.e. if funded, the firm contributes to precisely offset its production externality and non-shareholders are indifferent between contributing privately and not. So,

$$\begin{aligned} u\left(\frac{\pi(q^*) - \alpha^*}{n}, \alpha^* + (N-n)\gamma^* - \psi(q^*)\right) &= \frac{29}{9} \\ u\left(\frac{\pi(\hat{q})}{n}, -\psi(\hat{q})\right) &= \frac{4}{3} \end{aligned}$$

and thus

$$u\left(\frac{\pi(q^*) - \alpha^*}{n}, \alpha^* + (N-n)\gamma^* - \psi(q^*)\right) > u(1,0) > u\left(\frac{\pi(\hat{q})}{n}, -\psi(\hat{q})\right)$$

The left inequality confirms that the firm will be funded, but the right one violates Assumption 1.

We have shown that the firm will be funded, but we have not yet shown how much the funded firm will produce. The next lemma shows that since the public good is sufficiently desirable that individuals will make private contributions absent the firm, shareholders will optimally induce the manager of the firm to choose the socially optimal output.

Lemma 8 *If individuals are at least indifferent to contributing to the public good before the firm is funded, and the firm is funded, then the firm produces at the socially optimal level.*

The sketch of the proof (in the Appendix) is as follows. If this were not the case, then neither the manager nor shareholders would make contributions to the public good. However, when the firm is funded, then shareholders will have even stronger incentives to contribute to the public good than non-shareholders since they are wealthier. Moreover, non-shareholders have a stronger incentive to contribute to the public good than the ex ante situation since firm output reduces the level of the public good. As a result, if individuals are willing to contribute to the public good when the firm is absent, they will wish to contribute when the firm is operating as well. This implies that shareholders will optimally direct the manager to contribute to the public good on their behalf and the manager can do this most efficiently by choosing the socially optimal level of output.

Combining Lemmas 7 and 8 immediately implies Proposition 5. Now it is natural to ask how the *ex post* utilities of shareholders and non-shareholders compare. We will show that, if the bond pays more than the per shareholder, gross profits of the firm (producing the socially optimal quantity), then shareholders will always experience "buyer's remorse"—shareholders will be worse off than non-shareholders. Formally, the required condition is

Assumption 2: *The pollution free bond pays more than the firm, when it produces socially optimal quantity and contributes nothing to the public good:*

$$\frac{\pi\left(q^{*}\right)}{n}<1$$

Proposition 6 *Under Assumptions 1 and 2, (i) a firm producing the socially optimal quantity q^* will be formed, and (ii) shareholders will be worse off than non-shareholders.*

Proposition 6 is proved in the Appendix. Intuitively, when the pecuniary returns from the firm are lower than the bond, shareholders enjoy lower consumption levels than non-shareholders (since the manager is solving the free-rider problem on their behalf). Of course, all individuals enjoy the same level of public goods; therefore shareholders are worse off than non-investors *ex post*, but both groups are better off than if the firm is not funded.

To summarize, a common intuition suggests that socially responsible firms cannot arise if they disadvantage shareholders compared to other investment options. This intuition is flawed, because it only examines the *ex post* situation between shareholders and non-shareholders. But in selecting investments, individuals compare the *ex ante* situation, where the firm is not funded, to the *ex post* situation, where it is. If this comparison favors funding the firm, socially responsible firms can arise endogenously even in the presence of this *ex post* free-rider problem among investors.

5.2 Takeovers

Suppose we are in the situation described above: the funded firm produces at the socially optimal level, but shareholders earn less than non-shareholders. One might conjecture that the socially responsible firm makes an attractive takeover target. After all, the acquirer could always run the firm in a profit maximizing fashion to produce pecuniary returns in excess of the pollution free bond. Here we show that shareholders will spurn the offers of a profit maximizing takeover "artist."

Consider the ideal takeover artist—one with unlimited access to capital markets and no utility for public goods. Moreover, suppose that the takeover artist has all the bargaining power: if he can acquire even *one* share of the firm, then he can operate the firm as a pure profit maximizer. Our main result in this section is:

Proposition 7 *A socially responsible firm is immune to takeover by a profit maximizer.*

We will establish that, in the extreme situation where the takeover artist need only acquire a single share to run the firm as a profit maximizer, he cannot do so profitably. Obviously, this implies that when more shares are required or when the firm represents a mix of the motives of shareholders (i.e. it is not run purely as a profit maximizer), the takeover artist will also not find it profitable to acquire some or all of the firm.

Suppose that the takeover artist seeks to buy out shareholder 1. Let U^* denote the equilibrium utility of this shareholder. The takeover artist needs to choose a transfer t such that shareholder 1 is indifferent between selling out and retaining her share. If the shareholder accepts this offer, she earns

$$U' = u\left(t - \beta_1, \beta_1 + (n-1)\beta + (N-n)\gamma - \psi(\hat{q})\right)$$

where γ denotes the equilibrium contributions of non-shareholders, β_1 denotes the equilibrium contributions of shareholder 1, and β denotes the equilibrium contributions of all other shareholders. The most that the takeover artist can profitably offer to the shareholder is the full profits of the share when the firm is operated in a profit maximizing fashion, $t = \frac{\pi(\hat{q})}{n}$. In this case, shareholder 1 receives the same "dividend" as all of the shareholders remaining part of the firm, after it is run as a profit maximizer. Hence, the voluntary contributions of all shareholders (including the now-departed shareholder 1) are equal in equilibrium and then, shareholder 1's utility is at most

$$U' \leq u\left(\frac{\pi(\hat{q})}{n} - \beta, n\beta + (N-n)\gamma - \psi(\hat{q})\right)$$

However, since the manager of the firm (pre-takeover) selects output and voluntary contributions to maximize shareholder utility, we know that

$$u\left(\frac{\pi(\hat{q})}{n} - \beta, n\beta + (N-n)\gamma - \psi(\hat{q})\right) < u\left(\frac{\pi(q^*) - \alpha^*}{n}, \alpha^* + (N-n)\gamma^* - \psi(q^*)\right) = U^*$$

where γ^* denotes the equilibrium public goods contributions of non-shareholders when the firm is operated in a socially responsible fashion. This, though, implies that there is no transfer, t, that the takeover artist can offer a shareholder that both induces the shareholder to sell out and nets the takeover artist positive surplus. This proves Proposition 7.

Intuitively, since shareholders contract the manager to maximize their utility, the takeover artist cannot improve this situation for an individual shareholder by operating the firm differently. Moreover, since the takeover artist only enjoys returns in proportion to his holdings in the firm, he cannot cross-subsidize an individual shareholder by expropriating the remaining shareholders. Hence, a takeover artist cannot gain control of a socially responsible firm and earn positive rents.

5.3 Heterogeneous Shareholders

Up to now, we have assumed that shareholders have identical preferences and identical holdings. Under these assumptions, we showed that, when the public good is sufficiently desirable, firm output will be socially optimal, and shareholders will delegate all private public goods contributions to the manager. In this section, we investigate the extent to which these conclusions hold with heterogeneous shareholders. We have two primary findings. First, with suitable modification of the fundability condition, the manager continues to choose the socially optimal output level.

The obvious follow-up question is whether heterogeneity makes this condition more or less stringent compared to the identical shareholder case. That is, if we perturb the model, does the public good now have to be more or less socially desirable for the firm to still choose the socially optimally output. Our second main finding is that there is no general answer to this question. Depending on shareholder preferences, heterogeneity can make the fundability condition easier or harder to satisfy. We demonstrate this by assigning shareholders (commonly used) quasilinear utility functions, each with an exogenous parameter and then introduce heterogeneity such that the generalized mean of these parameters coincides with the parameter in the identical case—depending on which mean (e.g. geometric, arithmetic or quadratic) one introduces heterogeneity around, the stringency of the fundability condition can either increase or decrease.

Before proceeding, some preliminaries are in order. When shareholders are identical, the results of the first stage negotiations as to the manager's objective function are unambiguous. When shareholders differ, this is no longer the case. Differences in the rules used by shareholders in determining the manager's contract can lead to different objective functions on the part of the manager. For instance, a rule where each shareholder is given weight proportional to his or her ownership share will produce a share-weighted utilitarian objective function. A voting process might produce a contract whereby the manager is asked to maximize the utility of a *median* shareholder, whose identity might differ depending on who controls the agenda.[6] And so on.

Rather than taking a stand as to the exact process by which shareholders arrive at the manager's contract, we study a general, flexible form that nests the above approaches. Fix an increasing and concave aggregator function f, and suppose that shareholder i is entitled to a fraction λ_i (where $\sum \lambda_i \leq 1$) of net profits. The manager solves:

$$\max_{0\leq q, 0\leq \alpha \leq \pi(q)} f\left(u^1\left(c^1\left(q,\alpha\right), g\left(q,\alpha\right)\right), u^2\left(c^2\left(q,\alpha\right), g\left(q,\alpha\right)\right), ..., u^n\left(c^n\left(q,\alpha\right), g\left(q,\alpha\right)\right)\right) \quad (6)$$

where

$$\begin{aligned} c^i\left(q,\alpha\right) &= \lambda_i\left(\pi\left(q\right)-\alpha\right)-\beta^{i*}\left(q,\alpha\right) \\ g\left(q,\alpha\right) &= -\psi\left(q\right)+\alpha+\sum_{j=1}^{n}\beta^{j*}\left(q,\alpha\right) \end{aligned}$$

Here, $\beta^{i*}\left(q,\alpha\right)$ is the equilibrium individual contribution of shareholder i in voluntary contribution game when the manager chooses output q and contributes α to the public good. While the weights assigned to each shareholder are fixed from the manager's perspective following the first stage game, the same set of rules for determining this contract in the first stage can produce differing outcomes. For instance, a random proposal rule could produce different *ex post* weights depending on the identity of the shareholder with proposing power.

For the manager's problem to be well specified requires that we delineate the subsequent voluntary contributions that arise following any choice (q,α). The following lemma shows that there is a unique equilibrium following each such choice; hence, there is no ambiguity in the manager's problem.

Lemma 9 *Following any* (q,α), *there exists a unique equilibrium in the voluntary contributions game.*

Proof. Let β^i denote an equilibrium contribution by i in the voluntary contributions game. Suppose contrary to the lemma that, for some shareholder i, there exists an additional

[6] It is well-known that, in the context of multidimensional preferences, the median voter theorem is problematic. See, e.g., McKelvey 1976

equilibrium where voluntary contributions are $\beta^{i\prime} \neq \beta^i$. Let $g = \alpha + \sum_{i=1}^{n} \beta^i - \psi(q)$ be the public good provided in the original equilibrium and $g' = \alpha + \sum_{i=1}^{n} \beta^{i\prime} - \psi(q)$ denote the equilibrium level of the public good in an equilibrium where i contributes $\beta^{i\prime}$. ■

First, fix the manager's output, q, and consider voluntary contributions to the public good. When shareholders are identical, we showed that they optimally delegated *all* public goods provisioning to the manager. When shareholders differ, the result is weakened: shareholders who value public goods relatively more may privately contribute to the public good, even after the manager contributes. However, the broader intuition that the firm plays an important delegation role remains intact. Consistent with the identical shareholder case, the manager optimally increases the overall provisioning of the public good and leaves all shareholders better off than when they contribute only individually. Formally,

Proposition 8 *Under optimal manager contributions to the public good: (i) Not all shareholders contribute privately—at least one shareholder delegates all contributions to the manager. (ii) Overall public goods provisioning and shareholder welfare are higher relative to the case where the manager is barred from contributing.*

The proof (in the Appendix) resembles that of the homogenous case, Proposition 1. Limited delegation is a consequence of imperfect alignment between the manager's objectives and those of an *individual* shareholder. While the manager optimizes for some expression of collective preferences, an individual may care sufficiently about the public good that she continues to contribute privately. To completely resolve the free-riding problem for shareholders, a manager would have to be able to choose the amount of each shareholder's contribution to the public good from his individual share of profits, an unlikely possibility.

We now turn to the question of production quantities. Since the arguments in Proposition 2 did not rely on shareholders being identical, it follows immediately that:

Remark 1 *With heterogeneous shareholders, the firm produces strictly less than the profit maximizing quantity.*

But how much production does the firm undertake? It is intuitive (and may be readily verified) that the addition of shareholder heterogeneity leaves the socially optimal production level unchanged; thus, Lemma 4 continues to hold. We previously showed that the firm optimally chose the socially optimal level of production provided that the fundability condition held. Here, we amend the fundability condition to account for shareholder heterogeneity and show that it remains the case that the firm produces at the socially optimal level. As usual, we can again define *fundability* as the condition that manager contribution is not cornered at zero for the single production level q^*:

Condition 2 (Fundability) *We say the public good is* fundable *iff shareholders prefer the manager to contribute* something *to the public good when production is optimally abated. Formally,*

$$\sum_{i=1}^{n} \left(\lambda_i + \beta_\alpha^{i*}\right) u_c^i f_i \leq \left(1 + \sum_{i=1}^{n} \beta_\alpha^{i*}\right) \sum_{i=1}^{n} u_g^i f_i \tag{7}$$

where f_i, u_c^i, u_g^i and β_α^{i*} are evaluated at $\langle q, \alpha \rangle = \langle q^*, 0 \rangle$ for all i. Otherwise we say the public good is unfundable.

Define $\alpha(q)$ to be the manager's optimal contribution given output q. The following lemma (proved in the Appendix) implies that if the manager optimally makes strictly positive contributions to the public good at output q^*,then, when output is increased, so too are optimal managerial contributions to the public good. Formally,

Lemma 10 *Suppose that $\alpha(q') > 0$. Then, for all $q'' \in [q^*, \hat{q}]$, $\alpha(q'') > \alpha(q')$.*

Lemma 10 implies that, when output exceeds the social optimum, the manager can more efficiently provide the same level of public goods by reducing output and decreasing contributions to the public good. Thus, for the same reasons as when shareholders are identical, this implies that the manager will choose the socially optimal level of output. Proposition 9, which is proved in the Appendix, states this formally.

Proposition 9 *When shareholders are heterogeneous, the manager chooses the socially optimal quantity q^* iff the public good is* fundable *(in the sense of Condition 2) otherwise the* *firm overproduces (i.e. $q \in (q^*, \widehat{q})$). Furthermore, the manager provisions strictly positive amounts of the public good iff the inequality in equation (7) is strict.*

We can again make the policy comparison between a cleaner versus a more profitable production technology. Proposition 4 identified conditions in which there was neutrality between these two improvements. This result made no use of the fact that shareholders were identical; thus, it immediately extends to the heterogeneous case. We formalize this observation in the following remark.

Remark 2 *Suppose that Condition 2 holds. Then with heterogeneous shareholders, firm output is* identical *under the cleaner or cheaper technology. Furthermore, total public goods are* identical *under the two technology improvements.*

We now turn to the other primary question of this section: Does heterogeneity make *fundability* more or less difficult to satisfy? We will show that no general result along these lines exists. Suppose that preferences are quasi-linear of the following form:

$$U_i = c_i + \theta_i h(g)$$

where h is a strictly increasing and strictly concave function, and θ_i is an individual specific parameter capturing differing tastes for the public good. This functional form is useful in that it permits a simple comparison of a group of heterogeneous shareholders with an analogous set of identical "average" shareholders. To make this comparison we suppose that identical shareholders all have a θ parameter equal to the (generalized) mean of the θ_i parameters under heterogeneity. Recall that the generalized mean with parameter p of a list $\{\theta_i\}_{i=1}^n$ where each entry receives equal weight is simply

$$\bar{\theta}_p = \left(\sum \frac{1}{n} (\theta_i)^p\right)^{\frac{1}{p}}$$

It is well-known that $\bar{\theta}_p$ strictly increases in p. Setting $p = 1$ produces the usual arithmetic mean. Choosing other values of p produce geometric, harmonic, and other commonly used averages.

We can now compare the fundability condition under differing specifications of heterogeneity. When the manager is a utilitarian, the fundability condition for homogeneous shareholders is simply

$$nh'\left(-\psi\left(q^{*}\right)\right)\bar{\theta}_{p} \geq 1 \tag{8}$$

Now, if we introduce a small amount of heterogeneity around $\bar{\theta}_p$, such that no shareholder wishes to contribute privately, the analogous condition is

$$h'\left(-\psi\left(q^{*}\right)\right)\sum\theta_{i} \geq 1 \tag{9}$$

Proposition 10 *When shareholder have quasi-linear preferences with generalized mean $\bar{\theta}_p$ then:*

(i) When $p > 1$, fundability is less likely to be satisfied under heterogeneity.

(ii) When $p < 1$, fundability is more likely to be satisfied under heterogeneity.

(iii) When $p = 1$, fundability is satisfied under heterogeneity iff it is satisfied under homogeneity.

Proof. When $p = 1$, conditions (8) and (9) are identical, because

$$\frac{1}{n}\sum\theta_{i} = \bar{\theta}_{1}$$

Thus, heterogeneity is *neutral* with respect to fundability, and part *(iii)* of the proposition holds. Parts *(i)* and *(ii)* follow directly from the fact that $\frac{d\bar{\theta}_p}{dp} > 0$. ■

There is no particular justification for favoring any particular mean as being the "right" way to introduce heterogeneity. While the arithmetic mean is the most familiar form, it is purely arbitrary. Thus, we can conclude that heterogeneity has no systematic effect on fundability. While we have shown this for this simple form of quasi-linear preferences, the same effect holds more generally although the linkage to the mean of the θ parameters is specific to our functional form. We have shown that the local imposition of heterogeneity produces no systematic directional change in contributions by the manager. The same can be shown globally using numerical methods.

5.4 Competition

We have so far implicitly assumed that the firm is a monopoly. Suppose instead that it is one of K identical Cournot oligopolists. If one interprets the profit function $\pi(q)$ and externality function $\psi(q)$ as the residual profit and externality functions, then the proofs of the main propositions are unchanged. However, owing to strategic effects, total production in the market now exceeds the social optimum.

To see why, it is useful to add some structure to the model. As usual, profits depend on the total production in the market; however, since each competitor ignores the profit impact on the output of its competitors, we know that, for a given total output Q,then the monopolist's marginal profit $\pi'(Q) < \pi'_k(Q)$, the marginal profit of the kth Cournot competitor. There are many ways one could model the production externality. One simple way is to assume that it too only depends on total output in the market. In that case, the monopolist's marginal pollution cost, $\psi'(Q) = \psi'_k(Q)$, the marginal pollution cost of the kth Cournot oligopolist. The equality arises since each Cournot oligopolist views its output as marginal taking the outputs of the rivals as given. It then immediately follows that the total equilibrium output under Cournot competition

$$Q_C = \sum_{k=1}^{K} q_k^* > Q_M^*$$

Since each Cournot oligopolist, taking the output of the others as given, chooses its quantity so that the total output satisfies

$$\pi'_k(Q_C) = \psi'(Q_C)$$

and since $\pi'_k(\cdot) > \pi'(\cdot)$, it follows that

$$\pi'(Q_C) < \psi'(Q_C)$$

Hence, a monopolist (or equivalently a social planner) chooses a lower output.

To summarize, under imperfect competition, public goods contributions are still delegated to the manager. Total contributions to the public good still exceed the decentralized outcome, and production by each firm is less than the profit maximizing amount. However, overall production exceeds what is socially optimal.

6 Conclusion

One of the main functions of the Federal Trade Commission is to protect non-shareholding consumers from the actions of firms. Other agencies, such as the EPA, have similar roles. The perceived problem is that firms, in acting in the interests of shareholders, engage in actions contrary to, or at least ignorant of, the public good.

But what are the interests of shareholders? Economists usually assume that the goal of shareholders is pure profit maximization, and the main agency problem to be solved is properly incentivizing the manager of the firm to pursue this interest rather than indulging his or her tastes for other perks. Yet, shareholders have to breathe that same air as non-shareholders. They have to drink the same water. They have to look at the same blighted landscape stemming from firm production. In standard public economics models, individuals care both about private consumption as well as the public good, and the free-rider problem is the main obstacle to be overcome. The usual solution requires the intervention of the government, which can regulate production and tax so as to ensure the optimal provisioning of public goods for its citizens.

Since these same citizens are shareholders of firms, it stands to reason, that they might direct the manager of that firm to act in their interests—including their taste for public goods. While directing the manager to maximize profits *might* be consistent with this objective, perhaps the preferences of these shareholders could be better served by offering other incentives that account more fully for their tastes. In this paper, we study firm production accounting for shareholders' full range of tastes and preferences.

Our main finding is that, in general, shareholders will not opt for profit maximization but instead optimally direct the manager to pursue other goals as well. The manager serves an important commitment/coordination role by helping shareholders to solve the free-rider problem among themselves; thus, the manager will optimally be directed to divert firm profits towards contributions to the public good. More importantly, they will direct the manager *not* to pursue profit maximization in determining output. Indeed, our main result identifies conditions where shareholders will direct the firm to purse the *socially optimal* level of production, despite the fact that this could require a considerable sacrifice in profits. Broader society benefits too, though not by any design of the firm.

Thus, the firm offers another mechanism apart from government for ameliorating the free rider problem. Indeed, owing to its superior information about the trade-offs between profits and pollution as well as better information about shareholder preferences, the firm is conceivably more effective at this role than government. This is not to say that the firm perfectly solves the public goods problem. The need for governmental remedies remains; however, the amount of the public good increases compared to the case where firms engage in pure profit maximization and thus the scope for governmental intervention is lessened. Put differently, making managers more accountable to shareholders confers a *social* benefit.

Viewed through this lens, our results also rationalize a number of trends and empirical regularities. First, the recent increase in socially responsible business coincides with significant changes in the boardroom. In particular, shareholders in the US are now more active and powerful than at any time in the recent past. The model links the two trends—given the opportunity, shareholders will direct the firm to engage in socially responsible practices supporting their tastes for public goods. Second, empirical evidence suggests that the social behavior of a firm depends on the extent to which its owners and managers suffer harm or derive benefit. For example, Grant, Jones and Trautner (2004) find that absentee managed plants in the US emit more toxins, on average, than other plants. This too, is consistent with the model. In a sense, charity begins at home for shareholders—abatement activities will be greater to the extent that shareholders are directly affected by the emissions.

A number of other rationales have been offered to account for socially responsible behavior in firms. This behavior could be a consequence of agency problems. The implication here is that access to better contracting instruments or greater transparency as to the actions of a manager, then the behavior would disappear. This, however, is inconsistent with the observation that greater shareholder activism has coincided with *more* socially responsible behavior rather than less.

A more subtle argument is that these actions are a disguised form of profit maximization. In effect, socially responsible behavior is a differentiation strategy designed to capture value from socially-minded consumers. Yet little empirical evidence supports this view. Similarly, evidence that increased shareholder activism is generating higher profits, is weak. Our model offers a straightforward explanation for the absence of such findings—simply put, these behaviors are not profit maximization but rather utility maximization for shareholders with tastes valuing public goods as well as private consumption.

Under the disguised profit maximization hypothesis, the firm charges higher prices because consumers who value the actions of the firm now have higher willingness to pay. In our model, higher prices are not a response to consumer preferences, but rather a by-product of shareholder preferences. For instance, when shareholders feel the externalities of production and remote consumers do not, those consumers will (at least partially) pay for the externalities, because the supply reductions implied by Proposition 2 will increase the market clearing price. For example, if a US factory produces less pesticide because its US shareholders breathe the production exhaust, the reduced supply of pesticide will raise its cost. For consumers breathing the same air as shareholders, this trade-off may justify the additional price, but those a continent away will only lose consumer surplus.

Finally, socially responsible firms survive the model's "market test" as well. Even when potential shareholders can earn higher returns elsewhere, they will still willingly invest in a socially responsible firm owing to their tastes for public goods and resist the attempts to take over the firm and commit it to profit maximization. Moreover, this occurs even though, *ex post*, shareholders are worse off than those standing on the sidelines after the formation of the socially responsible firm. The reason is that, even though shareholders are worse off *ex post*, they are better off *ex ante*—forming the socially responsible firm ; that is, the formation of the socially responsible firm improves on the situation where no such firm exists.

This is not to say that ours is the only explanation for socially responsible business practices. Agency problems clearly play some role and the value of a firm's brand is clearly linked to its corporate conduct. Moreover, translating the diverse preferences of shareholders into managerial incentives is, in practice, a far more formidable hurdle than it is in the model. Nonetheless, our results provide a starting point for examining how the scope of shareholder preferences beyond merely making money impact business practices.

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A Appendix (Proofs)

Lemma 11 *Suppose that individual i is contributing $\beta^i > 0$ in an equilibrium. Then evaluated at her allocation (c_i, g)*

$$\begin{aligned} u_{cc} - u_{cg} &< 0 \\ u_{gg} - u_{cg} &< 0 \end{aligned}$$

Proof. Strictly convex preferences imply that shareholder utility functions satisfy diminishing marginal rates of substitution (MRS). Formally, diminishing MRS is equivalent to the condition that, for any allocation (c, g),

$$\frac{\partial MRS}{\partial c} = \frac{u_{cc}u_g - u_{cg}u_c}{(u_g)^2} < 0$$

Since $\beta^i > 0$, it then follows that $u_c = u_g$ and hence $u_{cc} - u_{cg} < 0$. The other inequality is obtained analogously by differentiating the MRS with respect to g ■

Lemma 1 *Following every (q, α), the voluntary contributions game has a unique, symmetric equilibrium.*

Proof. First, we show that any equilibrium is symmetric. Suppose to the contrary that shareholders i, j give different amounts $\beta_i < \beta_j$. Then, since these are optimizing, it follows that the optimality condition for shareholder i satisfies:

$$u_c\left(\frac{\pi(q) - \alpha}{n} - \beta_i, \alpha + \sum_{k=1}^{n} \beta_k - \psi(q)\right) = u_g\left(\frac{\pi(q) - \alpha}{n} - \beta_i, \alpha + \sum_{k=1}^{n} \beta_k - \psi(q)\right)$$

and likewise for j

$$u_c\left(\frac{\pi(q) - \alpha}{n} - \beta_j, \alpha + \sum_{k=1}^{n} \beta_k - \psi(q)\right) = u_g\left(\frac{\pi(q) - \alpha}{n} - \beta_j, \alpha + \sum_{k=1}^{n} \beta_k - \psi(q)\right)$$

But since $\beta_j > \beta_i$ and since $u_{cc} < 0$ then

$$\begin{aligned} u_c\left(\frac{\pi(q) - \alpha}{n} - \beta_j, \alpha + \sum_{k=1}^{n} \beta_k - \psi(q)\right) &> u_c\left(\frac{\pi(q) - \alpha}{n} - \beta_i, \alpha + \sum_{k=1}^{n} \beta_k - \psi(q)\right) \\ &= u_g\left(\frac{\pi(q) - \alpha}{n} - \beta_i, \alpha + \sum_{k=1}^{n} \beta_k - \psi(q)\right) \end{aligned}$$

But this is a contradiction. Therefore $\beta_i = \beta_j$ for all i, j in any equilibrium.

Now, we show uniqueness. Suppose to the contrary that there are two equilibrium giving levels, β and β' such that $\beta \neq \beta'$, where β (β') denotes the equilibrium contribution of every shareholder.

When $\beta, \beta' > 0$, then for each to comprise an equilibrium requires that

$$u_c\left(\frac{\pi(q) - \alpha}{n} - \beta, \alpha + n\beta - \psi(q)\right) = u_g\left(\frac{\pi(q) - \alpha}{n} - \beta, \alpha + n\beta - \psi(q)\right)$$

and similarly for β'. Treating β as a parameter and differentiating $u_c - u_g$, we obtain

$$\begin{aligned}\frac{\partial(u_c - u_g)}{\partial\beta} &= -u_{cc} + (n+1)\,u_{cg} - nu_{gg} \\ &= -(u_{cc} - u_{cg}) - n(u_{gg} - u_{cg})\end{aligned}$$

Recall from Lemma 11 that, at any equilibrium value of β, diminishing MRS implies that $(u_{cc} - u_{cg}) < 0$ and $(u_{gg} - u_{cg}) < 0$. This implies that, in the neighborhood of any equilibrium, $u_c - u_g$ is negative for slightly lower values of β and positive for higher values of β. Were there to be multiple equilibria, then at least one such equilibria must have the reverse sign in the neighborhood of the equilibrium point, but this is impossible.

It remains to deal with the case where one of the possible equilibrium points occurs at $\beta = 0$. If $\beta = 0$ comprises an equilibrium, then it must be that

$$u_c\left(\frac{\pi(q) - \alpha}{n} - \beta, \alpha - \psi(q) + n\beta\right)\Big|_{\beta=0} - u_g\left(\frac{\pi(q) - \alpha}{n} - \beta, \alpha - \psi(q)\right)\Big|_{\beta=0} > 0$$

From our previous analysis, we know that, at any interior equilibrium point, $u_c - u_g$ is negative for slightly lower values of β and positive for higher values of β. Since $u_c - u_g > 0$ at $\beta = 0$, then at the smallest $\beta > 0$ comprising an equilibrium, it must be that $u_c - u_g$ is positive for slightly lower values of β and negative for slightly higher values. But this is a contradiction. Therefore, if $\beta = 0$ is an equilibrium, it is the unique equilibrium, and similarly, if $\beta > 0$ is an equilibrium, then $\beta = 0$ is not an equilibrium.. ■

Lemma 3 *For every q, the manager's problem has a unique maximand in α.*

Proof. Fix q and define $\beta(\alpha)$ to be the equilibrium level of private contributions made by each shareholder following α. We will show that the shareholder's problem (which is equivalent to the manager's problem) is single peaked in α. Given a change in α, then, by Lemma 2, $\beta'(\alpha) \in \left\{0, -\frac{1}{n}\right\}$.

Recall that the manager's welfare function amounts to

$$W(\alpha) = u\left(\frac{\pi(q) - \alpha}{n} - \beta(\alpha), \alpha + n\beta(\alpha) - \psi(q)\right)$$

When $\beta(\alpha) = 0$, then $\beta'(\alpha) = 0$, and the manager's first order condition with respect to α is

$$W'(\alpha) = -\frac{1}{n}u_c + u_g = 0$$

Differentiating again

$$W''(\alpha) = u_{cc}\left(\frac{1}{n}\right)^2 - 2u_{cg}\frac{1}{n} + u_{gg}$$

Now, applying the approach in the proof of from Lemma 11, diminishing MRS implies that

$$\begin{aligned} u_{cc}u_g - u_c u_{cg} &< 0 \\ u_{gg}u_c - u_g u_{cg} &< 0 \end{aligned}$$

and now, substituting using the interior optimum condition, and simplifying we obtain

$$\frac{1}{n}u_{cc} - u_{cg} < 0$$

and similarly

$$u_{gg} - \frac{1}{n}u_{cg} < 0$$

Next, notice that $W''(\alpha)$ may be rewritten as

$$W''(\alpha) = \frac{1}{n}\left(\frac{1}{n}u_{cc} - u_{cg}\right) + \left(u_{gg} - \frac{1}{n}u_{cg}\right)$$

and since each of the terms in parentheses is negative, then the sum is negative as well. Thus, we have shown that in the neighborhood of any extrema, $W''(\alpha) < 0$.This implies that there is a unique extrema and moreover, that extreme point is a maximand. ■

Lemma 4 *The socially optimal production level, q^*, is the unique value of q solving $\pi'(q^*) = \psi'(q^*)$.*

Proof. First, consider the case where there is an interior solution for both q and α. In that case, the first order condition of the planner's problem (3) for α is

$$u_c = nu_g + \sum_{i=1}^{N-n} v_g^i \tag{10}$$

where v_g^i denotes the derivative with respect to its argument. The first order condition of the planner's problem for q is

$$\pi'(q)\, u_c = \psi'(q) \left(nu_g + \sum_{i=1}^{N-n} v_g^i \right) \tag{11}$$

Together, equations (10) and (11) imply that the socially optimal production quantity, q^*, satisfies $\pi'(q) = \psi'(q)$.

Next, we consider possible corner solutions. First, consider the case where $q = 0$. In that case, the constraint on α is binding and hence $\alpha = 0$. This is never optimal for the planner. If, instead, the planner produced an arbitrarily small amount, $q = \varepsilon$, and used the entire proceeds for the public good, i.e. $\alpha = \pi(\varepsilon)$ then welfare would be

$$nu\left(0, -\psi(\varepsilon) + \pi(\varepsilon)\right) + \sum_{i=1}^{N-n} v^i\left(-\psi(\varepsilon) + \pi(\varepsilon)\right) > nu(0,0) + \sum_{i=1}^{N-n} v^i(0)$$

where the inequality follows from the fact that $\pi'(0) > \psi'(0)$. The right-hand side of the above inequality is welfare under zero production; thus, $q = 0$ is never optimal, and equation (11) always holds with equality.

Finally, consider the case where $\alpha = \pi(q)$. The planner's problem then becomes

$$\max_q nu\left(0, -\psi(q) + \pi(q)\right) + \sum_{i=1}^{N-n} v^i\left(-\psi(q) + \pi(q)\right)$$

yielding the following first order condition for q

$$\left(-\psi'(q) + \pi'(q)\right) \left(nu_g + \sum_{i=1}^{N-n} v_g^i \right) = 0$$

Hence, the socially optimal production quantity satisfies $\pi'(q) = \psi'(q)$. Strict concavity of $\pi(\cdot)$ and strict convexity of $\psi(\cdot)$ imply that this solution is unique. ■

Proposition 3 *(i) The manager chooses the socially optimal quantity q^* iff the public good is fundable; otherwise the firm overproduces (i.e. $q \in (q^*, \widehat{q})$). (ii) Furthermore, the manager provisions strictly positive amounts of the public good iff the inequality in equation (4) is strict.*

Proof. Step 1: First, we will show that the manager chooses the socially optimal quantity q^* if the public good is *fundable*.

Step 1.a: We may use Proposition 1 to simplify the manager's problem to

$$\max_{0 \le q, 0 \le \alpha \le \pi(q)} u\left(\frac{\pi(q) - \alpha}{n}, -\psi(q) + \alpha\right) \tag{12}$$

Let q' and α' be the manager's optimal production quantity and public goods contribution decisions respectively. The (unconstrained) first order condition of the manager's problem (12) for α is

$$u_c = nu_g \tag{13}$$

and the (unconstrained) first order condition of the manager's problem for q is

$$\pi'(q')\, u_c = \psi'(q')\, nu_g \tag{14}$$

Together, equations (13) and (14) imply that, if an interior solution exists, the manager chooses production such that $\pi'(q) = \psi'(q)$; that is, he chooses the socially optimal production quantity, q^*. We now consider possible corners. There are three constraints: $q' \ge 0$, $\alpha' \ge 0$, and $\alpha' \le \pi(q')$.

Step 1.b: We will first establish that q is never cornered by showing that $q' \ge q^* > 0$. Suppose to the contrary that $q' < q^*$. This implies $\pi'(q') > \psi'(q')$. The manager could increase production by a sufficiently small ε, increase α by $\varepsilon \times \psi'(q')$ to completely offset the additional production externality, and increase the total dividends to shareholders by $\varepsilon \times (\pi'(q') - \psi'(q'))$. Since this deviation leaves shareholders strictly better off, it contradicts the optimality of q'. Furthermore, since $\pi'(0) > \psi'(0)$, $\pi(\cdot)$ is strictly concave and $\psi(\cdot)$ is strictly convex, we know $q^* > 0$. Thus, q is never cornered.

Step 1.c: We will show that if condition (4) is satisfied, then α is not cornered below. Observe that since $\frac{\partial u_c}{\partial q} < 0$ and $\frac{\partial u_g}{\partial q} > 0$ condition (4) implies

$$u_c\left(\frac{\pi(q')}{n}, -\psi(q')\right) \leq nu_g\left(\frac{\pi(q')}{n}, -\psi(q')\right) \tag{15}$$

for all $q' \geq q^*$. Furthermore, since $\frac{\partial u_c}{\partial \alpha} > 0$ and $\frac{\partial u_g}{\partial \alpha} < 0$, if condition (4) holds, there is always some $\alpha \geq 0$ large enough such that (13) will hold for any $q' \geq q^*$. Thus, if that α is not more than profits ($\alpha \leq \pi(q')$), then the solution is interior and by Step 1.a $q' = q^*$.

Step 1.d: We will show that when α is cornered above ($\alpha' = \pi(q')$), then $q' = q^*$. When the constraint $\alpha \leq \pi(q)$ binds ($\alpha = \pi(q)$), the manager's problem becomes

$$\arg\max_q u(0, -\psi(q) + \pi(q)) \tag{16}$$

with the first order condition for q

$$\pi'(q')\, u_g = \psi'(q')\, u_g$$

In this case, it is clear that $q' = q^*$. Thus, in Steps 1.a-1.d, we have shown that the manager chooses the socially optimal quantity q^* if the public good is *fundable*.

Step 2: Now we will show that if the manager chooses q^*, the public good is *fundable.* We do this by proving its contrapositve, namely, if the public good is *unfundable*, then $q' > q^*$. Suppose to the contrary that $q' \leq q^*$. From Step 1.b, it can only be the case that $q' = q^*$.

Step 2.a. We will show a contradiction when α is interior. In this case, the manager's first order condition for α must be met at $q' = q^*$:

$$u_c\left(\frac{\pi(q^*) - \alpha'}{n}, -\psi(q^*) + \alpha'\right) = nu_g\left(\frac{\pi(q^*) - \alpha'}{n}, -\psi(q^*) + \alpha'\right) \tag{17}$$

However, *unfundability* implies

$$\begin{aligned} u_c\left(\frac{\pi(q^*) - \alpha'}{n}, -\psi(q^*) + \alpha'\right) &> u_c\left(\frac{\pi(q^*)}{n}, -\psi(q^*)\right) \\ > \quad nu_g\left(\frac{\pi(q^*)}{n}, -\psi(q^*)\right) &> nu_g\left(\frac{\pi(q^*) - \alpha'}{n}, -\psi(q^*) + \alpha'\right) \end{aligned}$$

where the outer inequalities follow because $\frac{\partial u_c}{\partial \alpha} > 0$ and $\frac{\partial u_g}{\partial \alpha} < 0$, and this contradicts the manager's first order condition for α.

Step 2. b. We will show a contradiction when α is cornered below. In this case, the manager's first order condition cannot be met, because $u_c > nu_g$ for all $\alpha' \geq 0$. But then the manager's first order condition for q (eqn. (14)) cannot be met except at $q' > q^*$, a contradiction.

Step 2.c. We will show a contradiction when α is cornered above. In this case, the manager's first order cannot be met, because $u_c < nu_g$ for all $\alpha' \leq \pi(q^*)$, but *unfundability* means $u_c > nu_g$ for $\alpha' = 0$, a contradiction. Thus, in steps 2.a-2.c we have shown if the manager chooses q^*, the public good is *fundable.*

Step 3: Now we will show the manager provisions strictly positive amounts of the public good if the inequality in equation (4) is strict. A strict inequality in equation (4) implies *fundability*. From Step 1 $q' = q^*$ and either the manager's first order condition for α holds or α is bounded above. If α is bounded above we are done. Suppose contrary to α being interior, that $\alpha = 0$. Then the manager's first order condition for α implies

$$u_c\left(\frac{\pi(q^*)}{n}, -\psi(q^*)\right) \geq nu_g\left(\frac{\pi(q^*)}{n}, -\psi(q^*)\right)$$

but this contradicts the strict inequality in equation (4).

Step 4: Finally, we will show that if the manager provisions strictly positive amounts of the public good, the inequality in equation (4) is strict,. To do this we first argue that $\alpha' > 0$ implies that $q' = q^*$. From Step 1.a, it is sufficient to establish a contradiction when $q' > q^*$. Recall that this implies $\pi'(q') < \psi'(q')$. In that case, it is more efficient to reduce production than to pay for offsetting public goods—the manager could decrease production q by a sufficiently small ε, and because $\alpha > 0$, reduce α by $\varepsilon \times \psi'(q')$ resulting no net decrease in public goods, but increasing total dividends paid to shareholders by $\varepsilon \times (\psi'(q') - \pi'(q'))$, which contradicts the optimality of q'. Since optimal production occurs when $q = q^*$, equation (17) represents the manager's first order condition. But since $\frac{\partial u_c}{\partial \alpha} > 0$ and $\frac{\partial u_g}{\partial \alpha} < 0$ the following must hold if α where reduced to 0 :

$$u_c\left(\frac{\pi(q^*)}{n}, -\psi(q^*)\right) < nu_g\left(\frac{\pi(q^*)}{n}, -\psi(q^*)\right)$$

This is equation (4) where the inequality is strict. ∎

Proposition 4 *Suppose the fundability condition holds. Then firm output is identical under the cleaner or cheaper technology. Furthermore, total public goods are identical under the two technology improvements.*

Proof. Since fundability holds, marginal profits equals marginal externality. With the cleaner technology, there is a unique q solving

$$\hat{\psi}'(q) = \pi'(q) \tag{18}$$

while with the cheaper technology, there is a unique q' solving

$$\psi'(q') = \hat{\pi}'(q') \tag{19}$$

We will show that $q = q'$. Rewriting equation (5), we have

$$\pi'(q) - \hat{\psi}'(q) = \hat{\pi}'(q) - \psi'(q)$$

Next, notice from equation (18) that $\pi'(q) - \hat{\psi}'(q) = 0$; therefore, for this same value of q, it must be that $\hat{\pi}'(q) - \psi'(q) = 0$, which is the solution to equation (19). Hence, we may conclude that $q' = q$.

Next, notice that following the production decision, the manager uses α to balance consumption, c, and public goods provisioning, g. One can think of this as the manager's budget set. The trade-off between these is unaffected by the technology, so the slope of the manager's budget set is constant and identical under the two technology improvements.

Next, we will show that the budget sets themselves are identical. To see this, we will show that the two technology improvements lead to budget sets that share a common point. Specifically, notice that the maximum total public goods under the cleaner technology is $\pi(q^*) - \hat{\psi}(q^*)$ while the maximum under the cheaper technology is $\hat{\pi}(q^*) - \psi(q^*)$, where q^* solve equations (18) and (19). We claim that

$$\pi(q^*) - \hat{\psi}(q^*) = \hat{\pi}(q^*) - \psi(q^*)$$

Recall that

$$\pi(q^*) - \hat{\psi}(q^*) = \int_0^{q^*} \pi'(q) - \hat{\psi}'(q)\, dq$$

while

$$\hat{\pi}(q^*) - \psi(q^*) = \int_0^{q^*} \hat{\pi}'(q) - \psi'(q)\, dq$$

Differencing the two expressions, we obtain

$$\int_0^{q^*} \hat{\pi}'(q) - \pi'(q) - \left(\psi'(q) - \hat{\psi}'(q)\right) dq = 0$$

where the equality follows from equation (5). Since the budget sets are identical under the two technology improvements, the optimal choice must likewise be identical. Hence, total public goods are *identical* under the two technology improvements. ■

Lemma 5 *Suppose that the firm is operated purely to maximize profits, and all individuals contribute $\tilde{\gamma}$ to the public good. Under Assumption 1, individuals prefer to become shareholders rather than to invest in the pollution free bond. Formally,*

$$u\left(\frac{\pi(\hat{q})}{n} - \tilde{\gamma}, N\tilde{\gamma} - \psi(\hat{q})\right) > u(1 - \tilde{\gamma}, N\tilde{\gamma})$$

Proof. Recall from the Fundamental Theorem of Calculus (FTC) that

$$\begin{aligned} u\left(\frac{\pi(\hat{q})}{n} - \tilde{\gamma}, -\psi(\hat{q})\right) &= \int_0^{\tilde{\gamma}} -u_c\left(\frac{\pi(\hat{q})}{n} - x, -\psi(\hat{q})\right) dx + u\left(\frac{\pi(\hat{q})}{n}, -\psi(\hat{q})\right) \\ u(1 - \tilde{\gamma}, 0) &= \int_0^{\tilde{\gamma}} -u_c(1 - x, 0)\, dx + U(1, 0) \end{aligned}$$

Observe that $-u_c\left(\frac{\pi(\hat{q})}{n} - x, -\psi(\hat{q})\right) > -u_c(1 - x, 0)$ for all $x \geq 0$, because $\frac{\pi(\hat{q})}{n} > 1$ and $-\psi(\hat{q}) < 0$. Since $u\left(\frac{\pi(\hat{q})}{n}, -\psi(\hat{q})\right) > u(1, 0)$

$$u\left(\frac{\pi(\hat{q})}{n} - \tilde{\gamma}, -\psi(\hat{q})\right) > u(1 - \tilde{\gamma}, 0)$$

Applying the FTC again

$$\begin{aligned} u\left(\frac{\pi(\hat{q})}{n} - \tilde{\gamma}, N\tilde{\gamma} - \psi(\hat{q})\right) &= \int_0^{N\tilde{\gamma}} u_g\left(\frac{\pi(\hat{q})}{n} - \tilde{\gamma}, y - \psi(\hat{q})\right) dy + u\left(\frac{\pi(\hat{q})}{n} - \tilde{\gamma}, -\psi(\hat{q})\right) \\ u(1 - \tilde{\gamma}, N\tilde{\gamma}) &= \int_0^{N\tilde{\gamma}} u_g(1 - \tilde{\gamma}, y + 0)\, dy + u(1 - \tilde{\gamma}, 0) \end{aligned}$$

Observe that $u_g\left(\frac{\pi(\hat{q})}{n} - \tilde{\gamma}, -\psi(\hat{q}) + y\right) > u_g(1 - \tilde{\gamma}, 0 + y)$ for all $y \geq 0$, because $-\psi(\hat{q}) < 0$ and $\frac{\pi(\hat{q})}{n} - \tilde{\gamma} > 1 - \tilde{\gamma}$. Since $u\left(\frac{\pi(\hat{q})}{n} - \tilde{\gamma}, -\psi(\hat{q})\right) > u(1 - \tilde{\gamma}, 0)$ the result follows. ■

Lemma 6 *Under Assumption 1, total individual public goods contributions when the firm is funded and run as a pure profit maximizer exceed total individual public goods contributions when the firm is unfunded. Formally,*

$$n\hat{\beta} + (N - n)\,\hat{\gamma} > N\tilde{\gamma}$$

Proof. Consider the scenario where the firm is funded, run as a profit maximizer and giving is decentralized. Begin from a starting point where everyone contributes as they would were the firm unfunded ($\tilde{\gamma}$)—we will adjust contributions in several steps. Shareholders have more money ($\frac{\pi(\hat{q})}{n} - \tilde{\gamma} > 1 - \tilde{\gamma}$), non-shareholders have the same amount of money, and public goods levels are lower ($N\tilde{\gamma} - \psi(\hat{q}) < N\tilde{\gamma}$). Thus, every individual has incentive to contribute more, because each has a higher marginal utility for public goods than for consumption. Now, let all individuals increase contributions until non-shareholders' marginal utility of consumption equals their marginal utility of public goods, say $\overline{\gamma}$. Thus, $N\overline{\gamma} > N\tilde{\gamma}$. Since shareholders have more wealth, their marginal utility of consumption is lower—shareholders wish to contribute still more. When they do, non-shareholders will reduce their total contributions, but by an amount less than the additional amount contributed by shareholders because these reductions increase their wealth and thus reduce their marginal utility of consumption. Thus, $n\left(\hat{\beta} - \overline{\gamma}\right) > (N - n)\left(\overline{\gamma} - \hat{\gamma}\right)$. Together, the two inequalities imply,

$$n\hat{\beta} + (N - n)\,\hat{\gamma} > N\overline{\gamma} > N\tilde{\gamma}$$

which completes the proof. ■

The lemma immediately implies that even if one shareholder deviates from the equilibrium contribution, $\hat{\beta}$ and instead contributes $\tilde{\gamma}$, then overall private contributions to the public good will still be higher than the case where the firm is unfunded. Formally,

Corollary 1

$$\tilde{\gamma} + (n - 1)\,\hat{\beta} + (N - n)\,\hat{\gamma} > N\tilde{\gamma}$$

Proof. If a single shareholder individually contributes as if he held the bond instead, the progression of steps in the proof of Lemma 6 is unaffected. ■

Lemma 7 *Under Assumption 1, the firm will attract n shareholders to fund it. Formally,*

$$u\left(\frac{\pi(\bar{q})-\bar{\alpha}}{n},\bar{\alpha}+(N-n)\bar{\gamma}-\psi(\bar{q})\right)>u(1-\tilde{\gamma},N\tilde{\gamma})$$

where $\bar{q},\bar{\alpha}$ are the optimal production quantity and public goods contribution chosen by the manager and $\bar{\gamma}$ is the equilibrium individual contribution of the remaining individuals who hold bonds.

Proof. First, notice that

$$u\left(\frac{\pi(\bar{q})-\bar{\alpha}}{n},\bar{\alpha}+(N-n)\bar{\gamma}-\psi(\bar{q})\right)>u\left(\frac{\pi(\hat{q})}{n}-\hat{\beta},n\hat{\beta}+(N-n)\hat{\gamma}-\psi(\hat{q})\right)$$

because the manager optimally chooses $\bar{q}$ and $\bar{\alpha}$ to maximize shareholder utility (Recall that the inequality is strict even if no contributions to the public good were made in under either production level, because from Proposition 2 no properly incented manager would choose the profit maximizing quantity). Since when the firm maximizes profits and all contributions are decentralized each shareholder chooses $\hat{\beta}$ to maximize her utility

$$u\left(\frac{\pi(\hat{q})}{n}-\hat{\beta},n\hat{\beta}+(N-n)\hat{\gamma}-\psi(\hat{q})\right)>u\left(\frac{\pi(\hat{q})}{n}-\tilde{\gamma},\tilde{\gamma}+(n-1)\hat{\beta}+(N-n)\hat{\gamma}\right)$$

From Lemma 6

$$u\left(\frac{\pi(\hat{q})}{n}-\tilde{\gamma},\tilde{\gamma}+(n-1)\hat{\beta}+(N-n)\hat{\gamma}\right)>u\left(\frac{\pi(\hat{q})}{n}-\tilde{\gamma},N\tilde{\gamma}\right)>u\left(\frac{\pi(\hat{q})}{n}-\tilde{\gamma},N\tilde{\gamma}-\psi(\hat{q})\right)$$

From Lemma 5

$$u\left(\frac{\pi(\hat{q})}{n}-\tilde{\gamma},N\tilde{\gamma}-\psi(\hat{q})\right)>u(1-\tilde{\gamma},N\tilde{\gamma})$$

Thus, shareholders enjoy higher utility compared to the situation where the firm does not operate. ■

Lemma 8 *If individuals are at least indifferent to contributing to the public good before the firm is funded, and the firm is funded, then the firm produces at the socially optimal level.*

Proof. Suppose, contrary to our claim, that the firm does not produce at the socially optimal level. Clearly, the firm will not produce less than the socially optimal level; therefore, we need only consider the case where the firm produces more than the socially optimal level.

In that case, it follows from Proposition 3 that shareholders will also direct the manager to contribute nothing to the public good. We will show that this leads to a contradiction; formally, no $\alpha > 0$ can satisfy the manager's FOC over α

$$u_c\left(\frac{\pi(\bar{q})}{n}, (N-n)\bar{\gamma} - \psi(\bar{q})\right) > nu_g\left(\frac{\pi(\bar{q})}{n}, (N-n)\bar{\gamma} - \psi(\bar{q})\right)$$

where $\bar{q} \in [q^*, \hat{q})$ is the equilibrium production level and $\bar{\gamma}$ is the equilibrium private contribution of non-shareholders if the firm is funded. Since for any $q > 0$ the firm causes damage to public goods and investors contribute nothing to the cleanup, non-shareholders will contribute more ($\bar{\gamma} > \tilde{\gamma}$). Thus, non-investors' FOCs hold

$$u_c(1-\bar{\gamma}, (N-n)\bar{\gamma} - \psi(\bar{q})) = u_g(1-\bar{\gamma}, (N-n)\bar{\gamma} - \psi(\bar{q}))$$

but this also shows that non-shareholder will not, by themselves, bring total public goods back up to the pre-firm funding level—their marginal utility of consumption has increased with their increased giving ($1-\tilde{\gamma} > 1-\bar{\gamma}$).Thus, funding the firm drops overall public goods levels

$$N\tilde{\gamma} > (N-n)\bar{\gamma} - \psi(\bar{q})$$

In order for the firm to be funded, it must be that shareholders are better off *ex ante* funding the firm

$$u\left(\frac{\pi(\bar{q})}{n}, (N-n)\bar{\gamma} - \psi(\bar{q})\right) > u(1-\tilde{\gamma}, N\tilde{\gamma})$$

Thus, lower public goods implies shareholders must receive larger dividends than both *ex ante* bond holders and by extension *ex post* non-shareholders

$$\frac{\pi(\bar{q})}{n} > 1-\tilde{\gamma} > 1-\bar{\gamma}$$

But if investor dividends are higher than the consumption of non-shareholders and both groups enjoy the same level of public goods, then investors must contribute more than non-shareholders. However, if shareholders are willing to privately contribute strictly positive amounts to the public good, then they will optimally have the manager to make strictly positive contributions to the public good (Lemma 1). But this is a contradiction. ■

Proposition 6. *Under Assumptions 1 and 2, (i) a firm producing the socially optimal quantity q^* will be formed, and (ii) shareholders will be worse off than non-shareholders.*

Proof. Part (i) is merely a restatement of Proposition 5. To establish part (ii) let γ^* be the equilibrium contribution of non-shareholders.

Case 1 ($\gamma^* = 0$): From Assumption 2.

$$u\left(\frac{\pi(q^*) - \alpha^*}{n}, \alpha^* - \psi(q^*)\right) < u(1, \alpha^* - \psi(q^*))$$

The LHS is the payoff of the investor, and the RHS is the payoff of a non-shareholder.

Case 2 ($\gamma^* > 0$): Since shareholders privately contribute nothing to the public good it follows that

$$u_c\left(\frac{\pi(q^*) - \alpha^*}{n}, \alpha^* + (N-n)\gamma^* - \psi(q^*)\right) < u_g\left(\frac{\pi(q^*) - \alpha^*}{n}, \alpha^* + (N-n)\gamma^* - \psi(q^*)\right)$$

It then follows that, for all consumption levels $c \leq \frac{\pi(q^*)-\alpha^*}{n}$, then

$$u_c(c, \alpha^* + (N-n)\gamma^* - \psi(q^*)) < u_g(c, \alpha^* + (N-n)\gamma^* - \psi(q^*))$$

Since non-shareholders contribute strictly positive amounts, this implies that $1 - \gamma^* > \frac{\pi(q^*)-\alpha^*}{n}$, otherwise, they would not contribute. Hence

$$u\left(\frac{\pi(q^*) - \alpha^*}{n}, \alpha^* + (N-n)\gamma^* - \psi(q^*)\right) < u(1-\gamma^*, \alpha^* + (N-n)\gamma^* - \psi(q^*))$$

since utility is increasing in consumption. ■

Lemma 12 *Suppose that Assumption 1 holds. Then an equilibrium where n investors become shareholders and the remainder do not is trembling hand perfect.*

Proof. Index investors by i and assume that for all $i \leq n$ investors become shareholders. We will show that, when each investor trembles with probability ε,then the limit $\varepsilon \to 0$ of best responses converges to the actions specified above. Assume that if more than n investors choose to become shareholders, then only the investments of individuals with the lowest indices are accepted and the excess funds are returned to the remaining investors. Let $\bar{U}$

denote the utility to a shareholder when the firm is funded. Let $\bar{V}$ denote the utility to a non-shareholder when the firm is funded. Let $\tilde{U}$ denote the utility of an investor when the firm is not funded. Let C denote the number of individuals (other than i) who choose to become shareholders.

For $i \leq n$,the expected utility of becoming a shareholder are

$$\Pr[C \geq n-1] \bar{U} + \Pr[C < n-1] \tilde{U}$$

while the expected utility of not becoming a shareholder is

$$\Pr[C \geq n] \bar{V} + \Pr[C < n] \tilde{U}$$

Differencing these two expressions yields

$$\Delta = \left(\bar{U} - \tilde{U}\right) \Pr[C = n-1] - \left(\bar{V} - \bar{U}\right) \Pr[C \geq n]$$

If $\Delta \geq 0$ then it is a best response to become a shareholder; otherwise it is a best response not to become a shareholder. We previously established that $\bar{U} > \tilde{U}$;thus if $\bar{V} < \bar{U}$, then $\Delta > 0$.Otherwise, the best response depends on $\Pr[C = n-1]$ compared to $\Pr[C \geq n]$.

The behavior of investors $i > n$ is irrelevant. If they prefer to become shareholders, then the firm will be funded. If they prefer not to become shareholders, then the equilibrium is contingent on the behavior of investors $i \leq n$.

Thus, it suffices to show that $\lim_{\varepsilon \to 0} \Pr[C = n-1] = 1$ and $\lim_{\varepsilon \to 0} \Pr[C \geq n] = 0$. We will first compute the probability that exactly $n - 1 + q$ individuals contribute.

$$\sum_{i=0}^{n-1} \binom{n-1}{i} \binom{N-n}{n-1+q-i} (1-\varepsilon)^i \varepsilon^{(n-1)-i} (\varepsilon)^{n-1+q-i} (1-\varepsilon)^{(N-n)-(n-1+q-i)} \tag{20}$$

With the following simplification

$$\begin{aligned}
\binom{N-n}{n-1+q-i} &= \frac{(N-n)!}{\left(\prod_{j=1}^{q} (n-1+j-i)\right)(n-1-i)! \frac{((N-n)-(n-1-i))!}{\prod_{j=1}^{q} ((N-n)-(n-1-i)-q+j)}} \\
&= \binom{N-n}{n-1-i} \prod_{j=1}^{q} \left(\frac{(N-n)-(n-1-i)-q+j}{n-1+j-i} \right)
\end{aligned}$$

we can write (20)

$$\Pr\left[C=n-1+q\right]=\sum_{i=0}^{n-1}A\left(n-1,i\right)\left(\frac{\varepsilon}{1-\varepsilon}\right)^{q}\prod_{j=1}^{q}\frac{(N-n)-(n-1-i)-q+j}{n-1+j-i}$$

where $A\left(n-1,i\right)$ is simply the probability that exactly $n-1$ others contribute of which exactly i consist of individuals with low indices:

$$A\left(n-1,i\right)=\binom{n-1}{i}\binom{N-n}{n-1-i}\varepsilon^{2(n-1-i)}\left(1-\varepsilon\right)^{N-2n+1+2i}\left(\frac{\varepsilon}{1-\varepsilon}\right)^{q}$$

Hence

$$\Pr\left[C=n-1\right]=\sum_{i=0}^{n-1}A\left(n-1,i\right)$$

Taking limits as $\varepsilon\to0$, we obtain

$$\begin{aligned}\lim_{\varepsilon\to0}\Pr\left[C=n-1\right] &= \lim_{\varepsilon\to0}\sum_{i=0}^{n-1}\binom{n-1}{i}\binom{N-n}{n-1-i}\varepsilon^{2(n-1-i)}\left(1-\varepsilon\right)^{N-2n+1+2i}\\ &= \binom{n-1}{n-1}\binom{N-n}{n-1-(n-1)}\left(1\right)^{n-1}\left(0\right)^{0}\left(0\right)^{0}\left(1\right)^{(N-n)}\\ &= 1\end{aligned}$$

Next, notice that

$$\lim_{\varepsilon\to0}\left(\frac{\varepsilon}{1-\varepsilon}\right)^{q}\prod_{j=1}^{q}\left(\frac{(N-n)-(n-1-i)-q+j}{n-1+j-i}\right)=0$$

and hence

$$\begin{aligned}\Pr\left[C\geq n\right] &= \sum_{q=1}^{N-n}\Pr\left[C=n-1+q\right]\\ &= \sum_{q=1}^{N-n}\sum_{i=0}^{n-1}A\left(n-1,i\right)\left(\frac{\varepsilon}{1-\varepsilon}\right)^{q}\prod_{j=1}^{q}\frac{(N-n)-(n-1-i)-q+j}{n-1+j-i}\\ &= \sum_{i=0}^{n-1}A\left(n-1,i\right)\left(\sum_{q=1}^{N-n}\left(\frac{\varepsilon}{1-\varepsilon}\right)^{q}\prod_{j=1}^{q}\frac{(N-n)-(n-1-i)-q+j}{n-1+j-i}\right)\end{aligned}$$

and, taking limits

$$\lim_{\varepsilon\to0}\Pr\left[C\geq n\right]=0$$

which establishes that $\Delta>0$, which completes the proof. ∎

Proposition 8 *Under optimal manager contributions to the public good: (i) Not all shareholders contribute privately—at least one shareholder delegates all contributions to the manager. (ii) Overall public goods provisioning and shareholder welfare are higher relative to the case where the manager is barred from contributing.*

Proof. Suppose that the manager optimally contributes α'. Let β_i' be each shareholder's individual contribution to the public good, and let shareholder m be one individually *contributing the least*. Suppose contrary to the proposition that $\beta_m' > 0$ (i.e. the lowest contributor does not fully delegate). We will show that the manager can profitably deviate by increasing public goods provisioning and shareholder welfare such that shareholder m will optimally delegate all contributions to the manager. Let $c(q, \alpha', \beta_i')$ and $g(q, \alpha', \beta_i')$ be the levels of consumption and public goods respectively enjoyed by shareholder i. Suppose that the manager deviates by contributing $\alpha'' = \alpha' + n \times \beta_m'$ (i.e. an additional β_m' per shareholder) to the public good, then by Lemma 2, all shareholders would optimally respond by contributing $\beta_i' - \beta_m'$, and the overall level of the public good would be unchanged; that is, $c(q, \alpha'', \beta_i' - \beta_m') = c(q, \alpha', \beta_i')$ and $g(q, \alpha'', \beta_i' - \beta_m') = g(q, \alpha', \beta_i')$. Now suppose that the manager increased α'' slightly. Shareholder m is now at a corner solution; hence Lemma 2 no longer applies. Each shareholder would experience a decrease in utility of $u_c^i(c(q, \alpha', \beta_i'))$ and an increase of $n \times u_g^i(g(q, \alpha', \beta_i'))$ from increased public goods provision. But since each β_i' was optimal originally, then $u_c^i(c(q, \alpha', \beta_i')) = u_g^i(g(q, \alpha', \beta_i'))$, and hence $u_c^i(c(q, \alpha', \beta_i')) < n \times u_g^i(g(q, \alpha', \beta_i'))$. Thus, the manager's deviation increases the provisioning of the public good and the utility of every shareholder. Hence, it also increases the value of the manager's objective function, but this contradicts the notion that the original contract was optimal. ■

Lemma 10 *Suppose that $\alpha(q') > 0$. Then, for all $q'' \in [q^*, \hat{q}]$, $\alpha(q'') > \alpha(q')$.*

Proof. Let $\alpha' = \alpha(q')$ and $\alpha'' = \alpha(q'')$ be the optimal contributions given output q' (resp. q''). Define $c^i(q, \alpha, \beta^i) = \lambda_i(\pi(q) - \alpha) - \beta^i$ to be shareholder i's private consumption and $g(q, \alpha, \beta) = -\psi(q) + \alpha + \sum_{j=1}^n \beta^j$ be total public goods. Suppose that the manager increased production to $q'' = q' + \varepsilon$, where $\varepsilon > 0$ is sufficiently small. Holding managerial and private contribution fixed, *all* shareholders will consume more and enjoy less public goods. Since

$$\frac{u_g^i\left(c^i\left(q',\alpha',\beta^{i\prime}\right),g\left(q',\alpha',\beta'\right)\right)}{u_c^i\left(c^i\left(q',\alpha',\beta^{i\prime}\right),g\left(q',\alpha',\beta'\right)\right)}<\frac{u_g^i\left(c^i\left(q'',\alpha',\beta^{i\prime}\right),g\left(q'',\alpha',\beta'\right)\right)}{u_c^i\left(c^i\left(q'',\alpha',\beta^{i\prime}\right),g\left(q'',\alpha',\beta'\right)\right)}$$

for all i and positive contribution was optimal at q', more total contribution, either centralized or decentralized, is optimal at q''. Define $B(q,\alpha)$ to be the equilibrium set of shareholders who privately contribute or are precisely indifferent to contributing when the manager chooses production level q and public goods contribution α

$$B(q,\alpha)=\left\{i:u_c^i\left(c^i\left(q,\alpha,\beta^i\right),g\left(q,\alpha,\beta\right)\right)=u_g^i\left(c^i\left(q,\alpha,\beta^i\right),g\left(q,\alpha,\beta\right)\right)\right\}$$

Case 1: $B(q',\alpha')=\varnothing$. In this case, the additional contribution to the public good following output q'' must come from the manager, because no individual shareholder will contribute. Hence $\alpha''>\alpha'$.

Case 2: $B(q',\alpha')\neq\varnothing$. First, consider the response of shareholders to an increase in output from q' to q''. Each shareholder $i\in B(q',\alpha')$ would contribute more if no other shareholder $j\in B(q',\alpha')$ did, but since $\frac{d\beta^i}{d\beta^{j\neq i}}>-1$ every shareholder $i\in B(q',\alpha')$ will contribute more. So, all shareholders $i\in B(q',\alpha')$ would benefit from small increase above α'.

Now consider non-contributors: Even if all shareholders $i\in B(q',\alpha')$ spent all of their additional dividend on the public good (i.e. $\beta^{i\prime\prime}=\lambda_i\left(\pi\left(q''\right)-\pi\left(q'\right)\right)+\beta^{i\prime}$ for all $i\in B(q',\alpha')$ and $\beta^{i\prime\prime}=\beta^{i\prime}=0$ for all $i\notin B(q',\alpha')$), they could not recover the public good destroyed by the additional production externality because $\pi'(q'')<\psi'(q'')$. Since consumption would be identical (i.e. $c^i\left(q'',\alpha',\beta^{i\prime\prime}\right)=c^i\left(q',\alpha',\beta^{i\prime}\right)$) and public goods reduced (i.e. $g(q'',\alpha',\beta'')<g(q',\alpha',\beta')$) all contributing shareholders $i\in B(q',\alpha')$ are at a lower budget set. Thus, regardless of how contributing shareholders $i\in B(q',\alpha')$ (with convex preferences) increase their contribution, public goods decrease. Since it was optimal for the manager to contribute public goods at q', it must be optimal for the manager to contribute more to the public good now that public goods have decreased and the private consumption of all non-contributing shareholders $i\notin B(q',\alpha')$ has increased. Hence $\alpha''>\alpha'$. ■

The following Lemmas prove useful in proving Proposition 9.

Lemma 13 *The relationship between output, individual, and managerial contributions to the public good satisfies*

$$\begin{aligned}
\pi'(q)\,\beta^i_\alpha(q,\alpha) &< -\beta^i_q(q,\alpha) < \psi'(q)\,\beta^i_\alpha(q,\alpha) \iff q < q^* \\
\pi'(q)\,\beta^i_\alpha(q,\alpha) &= -\beta^i_q(q,\alpha) = \psi'(q)\,\beta^i_\alpha(q,\alpha) \iff q = q^* \\
\pi'(q)\,\beta^i_\alpha(q,\alpha) &> -\beta^i_q(q,\alpha) > \psi'(q)\,\beta^i_\alpha(q,\alpha) \iff q > q^*
\end{aligned}$$

Proof. Given that the manager produces q and contributes α to the public good and that other shareholders contribute β_{i-}, shareholder i's optimality condition balances her marginal utilities of consumption and public good

$$\begin{aligned}
0 = F_i \equiv & -u^i_c\left(\lambda_i(\pi(q)-\alpha)-\beta^i, \alpha+\sum_{j=1}^n \beta^j-\psi(q)\right) \\
& +u^i_g\left(\lambda_i(\pi(q)-\alpha)-\beta^i, \alpha+\sum_{j=1}^n \beta^j-\psi(q)\right)
\end{aligned}$$

From the Implicit Function Theorem

$$\beta^i_q(q,\alpha) = -\left(D_\beta F^{-1}\right)_i \cdot D_q F \tag{21}$$

$$\beta^i_\alpha(q,\alpha) = -\left(D_\beta F^{-1}\right)_i \cdot D_\alpha F \tag{22}$$

where the i^{th} coordinate of the column vectors D_qF and $D_\alpha F$ respectively are

$$(D_qF)_i = \frac{dF_i}{dq} = -\left(\pi'(q)\,\lambda_i\left(u^i_{cc}-u^i_{cg}\right)+\psi'(q)\left(u^i_{gg}-u^i_{cg}\right)\right) \tag{23}$$

$$(D_\alpha F)_i = \frac{dF_i}{d\alpha} = \lambda_i\left(u^i_{cc}-u^i_{cg}\right)+\left(u^i_{gg}-u^i_{cg}\right) \tag{24}$$

Note that $\frac{\partial F_i}{\partial \beta_i} = u^i_{cc}-u^i_{cg}+u^i_{gg}-u^i_{cg}$ and $\frac{\partial F_i}{\partial \beta_j} = u^i_{gg}-u^i_{cg}$ when $i \neq j$. Since from Lemma 11 $u^i_{cc}-u^i_{cg}<0$ and $u^i_{gg}-u^i_{cg}<0$, the rows of $D_\beta F$ are linearly independent. Thus, $D_\beta F$ is invertible. From (21)-(24)

$$\begin{aligned}
-\beta^i_q(q,\alpha) > \psi'(q)\,\beta^i_\alpha(q,\alpha) \iff & \ \pi'(q)\,\lambda_i\left(u^i_{cc}-u^i_{cg}\right)+\psi'(q)\left(u^i_{gg}-u^i_{cg}\right) \\
> & \ \psi'(q)\left(\lambda_i\left(u^i_{cc}-u^i_{cg}\right)+\left(u^i_{gg}-u^i_{cg}\right)\right) \\
\iff & \ \pi'(q) < \psi'(q) \\
\iff & \ q > q^*
\end{aligned}$$

and similarly

$$
\begin{aligned}
\pi'(q)\,\beta^i_\alpha(q,\alpha) > -\beta^i_q(q,\alpha) \quad &\Longleftrightarrow \quad \pi'(q)\left(\lambda_i\left(u^i_{cc}-u^i_{cg}\right)+\left(u^i_{gg}-u^i_{cg}\right)\right) \\
&\quad > \quad \pi'(q)\,\lambda_i\left(u^i_{cc}-u^i_{cg}\right)+\psi'(q)\left(u^i_{gg}-u^i_{cg}\right) \\
&\Longleftrightarrow \quad \pi'(q)<\psi'(q) \\
&\Longleftrightarrow \quad q>q^*
\end{aligned}
$$

■

Proposition 9 *The manager chooses the socially optimal quantity q^* iff the public good is fundable; otherwise the firm overproduces (i.e. $q \in (q^*, \widehat{q})$). Furthermore, the manager provisions strictly positive amounts of the public good iff the inequality in equation (7) is strict.*

Proof. Step 1: First, we will show that the manager chooses the socially optimal quantity q^* if the public good is *fundable.*

Step 1.a: Let q' and α' be the manager's optimal production quantity and public goods contribution decisions respectively. The (unconstrained) first order condition of the manager's problem (6) for α is

$$\sum_{i=1}^{n}\left(\lambda_i+\beta^{i*}_\alpha(q',\alpha')\right)u^i_c f_i=\left(1+\sum_{j=1}^{n}\beta^{j*}_\alpha(q',\alpha')\right)\sum_{i=1}^{n}u^i_g f_i \tag{25}$$

and the (unconstrained) first order condition of the manager's problem for q is

$$\sum_{i=1}^{n}\left(\lambda_i\pi'(q')-\beta^{i*}_q(q',\alpha')\right)u^i_c f_i=\left(\psi'(q')-\sum_{j=1}^{n}\beta^{j*}_q(q',\alpha')\right)\sum_{i=1}^{n}u^i_g f_i \tag{26}$$

From Lemma 13, and (26) we know

$$
\begin{aligned}
\pi'(q')\sum_{i=1}^{n}\left(\lambda_i+\beta^{i*}_\alpha(q',\alpha')\right)u^i_c f_i &< \psi'(q')\left(1+\sum_{j=1}^{n}\beta^{j*}_\alpha(q',\alpha')\right)\sum_{i=1}^{n}u^i_g f_i \Longleftrightarrow q'<q^* \\
\pi'(q')\sum_{i=1}^{n}\left(\lambda_i+\beta^{i*}_\alpha(q',\alpha')\right)u^i_c f_i &= \psi'(q')\left(1+\sum_{j=1}^{n}\beta^{j*}_\alpha(q',\alpha')\right)\sum_{i=1}^{n}u^i_g f_i \Longleftrightarrow q'=q^* \\
\pi'(q')\sum_{i=1}^{n}\left(\lambda_i+\beta^{i*}_\alpha(q',\alpha')\right)u^i_c f_i &> \psi'(q')\left(1+\sum_{j=1}^{n}\beta^{j*}_\alpha(q',\alpha')\right)\sum_{i=1}^{n}u^i_g f_i \Longleftrightarrow q'>q^*
\end{aligned}
$$

Substituting (25) into the above implies that one or more of the following must hold at the optimal q :

$$\begin{aligned}
\pi'(q') &< \psi'(q') \iff q' < q^* \\
\pi'(q') &= \psi'(q') \iff q' = q^* \\
\pi'(q') &> \psi'(q') \iff q' > q^*
\end{aligned}$$

The first and last statements are false. Thus, if an interior solution exists, it can only be at at $q' = q^*$. We now consider possible corners. There are three constraints: $q' \geq 0$, $\alpha' \geq 0$, and $\alpha' \leq \pi(q')$.

Step 1.b: We will first establish that q is never cornered by showing that $q' \geq q^* > 0$. Suppose to the contrary that $q' < q^*$. This implies $\pi'(q') > \psi'(q')$. The manager could increase production by a sufficiently small positive ε, increase α by $\varepsilon \times \psi'(q')$ to completely offset the additional production externality, and increase the total dividends to shareholders by $\varepsilon \times (\pi'(q') - \psi'(q'))$. Since this deviation leaves all shareholders better off, it contradicts the optimality of q'. Furthermore, since $\pi'(0) > \psi'(0)$, $\pi(\cdot)$ is strictly concave and $\psi(\cdot)$ is strictly convex, we know $q^* > 0$. Thus, q is never cornered.

Step 1.c. We will show that if the condition (7) is satisfied, then α is not cornered below. If the condition (7) is satisfied and the manager were to chose production level q^*, shareholders would wish the manager to contribute something to the public good. In turn, then from Lemma 10 we know that the shareholders would like the manager to contribute something for all $q' \geq q^*$— in other words, the manager's first order condition for α (13) can only be satisfied by a sufficiently large nonnegative α. Thus, if this α is not more than profits ($\alpha \leq \pi(q')$), then the solution is interior and by Step 1.a. $q' = q^*$.

Step 1.d. We will show that when α is cornered above ($\alpha' = \pi(q')$, $\beta^{i*}(q', \pi(q')) = 0$), then $q' = q^*$. When the constraint $\alpha \leq \pi(q)$ binds ($\alpha = \pi(q)$) the manager's problem becomes

$$\max_{0 \leq q} f\left(u^1(0, -\psi(q) + \pi(q)), u^2(0, -\psi(q) + \pi(q)), ..., u^n(0, -\psi(q) + \pi(q))\right) \tag{27}$$

with the first order condition for q

$$\pi'(q)\sum_{i=1}^{n} f_i u_g^i = \psi'(q)\sum_{i=1}^{n} f_i u_g^i$$

In this case, it is clear that $q' = q^*$. Thus, in Steps 1.a - 1.d, we have shown that the manager chooses the socially optimal quantity q^* if the public good is *fundable*.

Step 2: Now we will show that if the manager chooses q^*, the public good is *fundable*. We do this by proving its contrapositive, namely, if the public good is *unfundable*, then $q' > q^*$. Suppose to the contrary that $q' \leq q^*$. From Step 1.b, it can only be that the case that $q' = q^*$.

Step 2.a: We will show a contradiction when α is interior. Since q^* is optimal the manager's first order condition for q (eqn. 26) must hold at q^* and optimal provisioning α'

$$\sum_{i=1}^{n}\left(\lambda_i \pi'(q^*) - \beta_q^{i*}(q^*, \alpha')\right) u_c^i f_i = \left(\psi'(q^*) - \sum_{j=1}^{n}\beta_q^{j*}(q^*, \alpha')\right)\sum_{i=1}^{n} u_g^i f_i$$

From Lemma 13 this can be rewritten

$$\pi'(q^*)\sum_{i=1}^{n}\left(\lambda_i + \beta_\alpha^i(q^*, \alpha')\right) u_c^i f_i = \psi'(q^*)\left(1 + \sum_{j=1}^{n}\beta_\alpha^{j*}(q, \alpha)\right)\sum_{i=1}^{n} u_g^i f_i$$

where $\pi'(q^*) = \psi'(q^*)$. However, *unfundability* implies that for all $\alpha' \geq 0$

$$\sum_{i=1}^{n}\left(\lambda_i + \beta_\alpha^{i*}(q^*, \alpha')\right) u_c^i f_i > \left(1 + \sum_{j=1}^{n}\beta_\alpha^{j*}(q^*, \alpha')\right)\sum_{i=1}^{n} u_g^i f_i$$

which is a contradiction.

Step 2.b: We will show a contradiction when α is cornered below. In this case, the manager's first order condition for α cannot be met, because

$$\sum_{i=1}^{n}\left(\lambda_i + \beta_\alpha^{i*}(q^*, \alpha')\right) u_c^i f_i > \left(1 + \sum_{i=1}^{n}\beta_\alpha^{i*}(q^*, \alpha')\right)\sum_{i=1}^{n} u_g^i f_i \tag{28}$$

for all $\alpha' \geq 0$. Since q^* is optimal, the manager's first order condition for q must be satisfied. Using Lemma 2 it can be written

$$\pi'(q^*)\sum_{i=1}^{n}\left(\lambda_i + \beta_\alpha^{i*}(q^*, \alpha')\right) u_c^i f_i = \psi'(q^*)\left(1 + \sum_{j=1}^{n}\beta_\alpha^{j*}(q^*, \alpha')\right)\sum_{i=1}^{n} u_g^i f_i \tag{29}$$

But from (28) this cannot be met except at $q' > q^*$, a contradiction.

Step 2.c. We will show a contradiction when α is cornered above. In this case, the manager's first order condition cannot be met, because

$$\sum_{i=1}^{n}\left(\lambda_i+\beta_{\alpha}^{i*}\left(q^*,\alpha'\right)\right)u_c^i f_i<\left(1+\sum_{j=1}^{n}\beta_{\alpha}^{j*}\left(q^*,\alpha'\right)\right)\sum_{i=1}^{n}u_g^i f_i$$

for all $\alpha' \leq \pi\left(q^*\right)$. but *unfundability* means

$$\sum_{i=1}^{n}\left(\lambda_i+\beta_{\alpha}^{i*}\left(q^*,0\right)\right)u_c^i f_i>\left(1+\sum_{j=1}^{n}\beta_{\alpha}^{j*}\left(q^*,0\right)\right)\sum_{i=1}^{n}u_g^i f_i$$

for $\alpha' = 0$, a contradiction. Thus, in Steps 2.a - 2.c we have shown that if the manager chooses q^*, the public good is fundable.

Step 3: Now we will show the manager provisions strictly positive amounts of the public good if the inequality in equation (7) is strict. A strict inequality in equation (7) implies fundability. From Step 1 $q' = q^*$ and either the manager's first order condition for α holds or α is bounded above. If α is bounded above we are done. Suppose contrary to being interior, that $\alpha = 0$. Then the manager's first order condition for α implies

$$\sum_{i=1}^{n}\left(\lambda_i+\beta_{\alpha}^{i*}\left(q^*,\alpha'\right)\right)u_c^i f_i\geq\left(1+\sum_{j=1}^{n}\beta_{\alpha}^{j*}\left(q^*,\alpha'\right)\right)\sum_{i=1}^{n}u_g^i f_i$$

but this contradicts the strict inequality of (7).

Step 4: Finally, we show that if the manager provisions strictly positive amounts of the public good, the inequality in equation (7) is strict. To do this first observe that $\alpha' > 0$ means that the manager's first order condition for α is met (i.e. α is interior) or α is bounded above. From the arguments of Step 1 we know we that $q' = q^*$.Having established that optimal production occurs when $q = q^*$, we will show a contraction when $\alpha' > 0$ and the the inequality in (7) is not strict. Suppose, contrary to the proposition, that the fundability condition is tight (recall that the case where the equality in the above is "greater than" (>) has already been ruled out in Step 2).

$$\sum_{i=1}^{n}\left(\lambda_i+\beta_{\alpha}^{i*}\left(q^*,0\right)\right)u_c^i f_i=\sum_{i=1}^{n}\left(1+\sum_{j=1}^{n}\beta_{\alpha}^{j*}\left(q^*,0\right)\right)u_g^i f_i \tag{30}$$

This is precisely the manager's first order condition for α when $\alpha' = 0$, but this contradicts the predicate that optimal $\alpha' > 0$. ■